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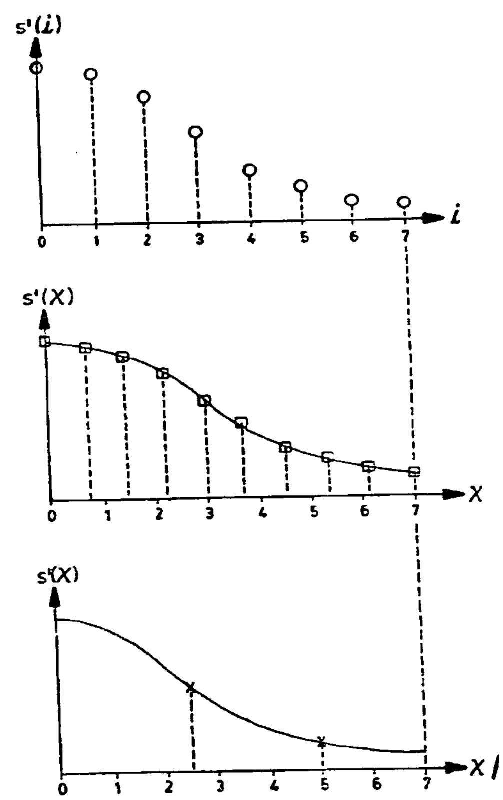
(54) Title: CODING METHODS AND APPARATUS FOR SCALING AND FILTERING IMAGES USING DISCRETE COSINE TRANSFORMS

(57) Abstract

Image processing method and apparatus by which images in the spatial domain can be represented in the frequency domain through the use of discrete cosine transforms, conveniently operated on to achieve scaling and filtering effects while in the frequency domain, and then retransformed to the spatial domain or stored, displayed, reproduced or transmitted to distant locations for subsequent reuse. The scaling techniques which the invention utilizes can be for image enlargement by interpolation or image reduction by decimation. In the case of decimation, a filtering operation preferably is first performed in the frequency domain to avoid artifacts in the decimation process. The filtering operation is mathematically equivalent to a linear convolution in the spatial domain as a consequence of the properties of the DCT transformation. In both interpolation and decimation procedures, use is made of a hybrid inverse discrete cosine transform in which the argument of the series of cosine terms are evaluated at values arrived at by scaling ratio considerations, rather than the usual sampling index increments. As a consequence, image points other than original image data in the spatial domain can be created or replaced by an approximation technique which involves representing them in frequency space by a series of terms that can be considered to be continuous over the range of the sampling index corresponding to that of the original image data.



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CODING METHODS AND APPARATUS FOR SCALING AND FILTERING IMAGES USING DISCRETE COSINE TRANSFORMS

CROSS REFERENCE

This application is related to United States patent application Serial Number 761,660, now U.S. Patent No. 5,168,375.

BACKGROUND OF THE INVENTION

1. Field of the Invention

This invention relates generally to improved methods and apparatus for image processing. More particularly, the invention relates to novel methods and apparatus for using discrete cosine transformations to: enlarge an image by interpolation; reduce an image by decimation; and/or filter an image in the frequency domain by a method equivalent to a mathematical convolution in the spatial domain.

2. Description of the Prior Art

Images can be thought of as two-dimensional representations of some visual reality that is distributed in space and/or time. Ordinarily, they are what the human visual system perceives as variations in external stimuli such as brightness, color, and sometimes depth cues. While over the years any number of techniques have been developed to capture and reproduce images, their represention as continuous, discrete, or digital signals which can be manipulated, processed or displayed through the use of computers or other special purpose electronic hardware is the most recent technique, now well-established, which has a variety of beneficial applications. For instance, while in electronic form,

images can be enhanced to create special visual effects, restored, coded for transmission to distant locations, reconstructed, displayed, or converted to some other tangible form.

Processing an electronic image may include such operations as filtering, sharpening, smoothing, compressing, decompressing, enlarging, reconstructing, and printing the image, or any portion thereof, in various image processing systems such as an electronic camera, camcorder, printer, computer or any other imaging device.

Image processing can occur in either the spatial domain or the frequency domain. An image is said to reside in the spatial domain when the values of the parameters used to describe it, such as brightness, have a direct correspondence with spatial location. In the frequency domain, the image in the spatial domain may be represented by a series of frequency components in the form of trigonometric functions which, when summed for each image point (i.e., pixel) yield the value of the parameter used to characterize the image of that point in the spatial domain, and such a representation may be extended to cover all points of an image.

In the spatial domain, original image data may be conveniently represented as image data points in a first spatial matrix designated, $s(j,i)$, for a two-dimensional case where the lower case, s, designates the spatial domain, i is the index of rows and j is the index of columns. In the frequency domain, matrices can also be used to mathematically describe an image as a set of the transform coefficients (also referred to as frequency coefficients) which represent frequency data in a transform matrix conventionally designated, $S(v,u)$, where the upper case, S, designates the frequency domain and, u is the number of rows and v is the number of columns.

Spatial image data points may be transformed to frequency space using transformations such as Fourier transforms or discrete cosine transforms (DCTs). When the transformation involved is a discrete cosine transformation, the frequency domain is referred to as the DCT domain and the frequency

coefficients are referred to as DCT coefficients. Conventionally, transforming data from the spatial domain to the frequency domain is referred to as a forward transformation, whereas transforming data from the frequency domain to the spatial domain is referred to as an inverse transformation. Hence, a forward discrete cosine transformation is defined as a transform that maps an image from the original image data points s(j,i) in the spatial domain to DCT coefficients S(v,u) in the DCT domain according to the basis function of the forward DCT, whereas an inverse discrete cosine transformation (or IDCT) is defined as a transform that maps the DCT coefficients S(v,u) from the DCT domain to reconstructed image data points s'(j,i) in the spatial domain according to the basis function of the IDCT.

The use of DCT and IDCT transforms for compressing or decompressing images to reduce memory storage requirements and/or increase transfer and computational speeds is well-known and, in fact, the practice has been adopted as standard in industry by such groups as The Joint Photographic Experts Group (JPEG), which was created as part of a joint effort of the Consultative Committee on International Telegraphy and Telephony (CCITT) and The International Standards Organization (ISO). Today, most image processing programs support both loading and saving files that conform to JPEG standards for files or formats, and there is even custom hardware on the market for compressing and decompressing in JPEG format.

Even so, the application of DCT transforms to image processing operations other than compression is not well-known and offers an opportunity to conveniently utilize available custom hardware or general purpose computers to achieve other processing effects such as scaling, filtering, or the conditioning of image data for efficient transmission while suppressing or reducing artifacts.

Consequently, it is a primary object of this invention to provide methods and apparatus which utilize discrete cosine tranforms for image processing operations other than compression.

It is another object of the present invention to provide methods and apparatus which utilize discrete cosine transforms for enhancing compression while reducing artifacts.

Other objects of the invention will, in part, appear hereinafter and, in part, be obvious when the following detailed description is read in conjunction with the drawings.

SUMMARY OF THE INVENTION

Image processing methods and apparatus by which images in the spatial domain can be represented in the frequency domain through the use of discrete cosine transforms, conveniently operated on to achieve scaling and filtering effects while in the frequency domain, and then retransformed to the spatial domain or stored, displayed, reproduced or transmitted to distant locations for subsequent reuse.

The scaling techniques which the invention utilizes can be for image enlargement by interpolation or image reduction by decimation. In the case of decimation, a filtering operation preferably is first performed in the frequency domain to avoid artifacts in the decimation process. The filtering operation is mathematically equivalent to a linear convolution in the spatial domain as a consequence of the properties of the DCT transformation.

In both interpolation and decimation procedures, use is made of a hybrid inverse discrete cosine transform in which the argument of the series of cosine terms are evaluated at values arrived at by scaling ratio considerations rather than the usual sampling index increments. As a consequence, image points other than those comprising original image data in the spatial domain can be generated by an approximation technique which involves representing them in frequency space by a series of terms that can be considered to be continuous over the range of the sampling index corresponding to that of the original image data.

In use, the hybrid technique involves first transforming spatial image data to frequency space through the use of a standard DCT. Here, a DCT basis

matrix and its transpose for two-dimensional case are matrix multiplied by the image data in matrix form to generate a matrix of DCT coefficients. A scaling ratio is then chosen and a hybrid IDCT basis matrix is generated along with its transpose. The results of these two operations are mathematically combined to generate a reconstructed image data matrix through an inverse transform step. The reconstructed image data matrix represents the new image which may be either enlarged or reduced in size.

The scaling operations can be carried out so that the magnification is the same along orthogonal azimuths or different along both since the operations can be mathematically decoupled by reason of the orthogonal property of the DCT basis functions.

To perform the filtering operation for sharpening or smoothing, use is made of a discrete odd cosine transform of a symmetric filtering kernel in the spatial domain. When decimating, the filtering operation is preferably performed to avoid artifacts and adjacent image data matrices are preferably overlapped by at least one row and column.

All operations are preferably performed in JPEG format and special purpose hardware adapted to the JPEG standard may also be beneficially employed to process image data in 8X8 or 16X16 sized blocks.

BRIEF DESCRIPTION OF THE DRAWINGS

The aforementioned aspects and other features of the invention are described in detail in conjunction with the accompanying drawings in which the same reference numerals are used throughout for denoting corresponding elements and wherein:

Fig. 1 is a plot of discrete cosine normalized basis functions for the one dimensional case of a DCT as given by equation (1);

Fig. 2A is a graph of reconstructed image data points $s'(i)$ vs. i for an eight element one dimensional matrix with a 1:1 scaling ratio;

Fig. 2B is a graph (related to image enlargement) of reconstructed image data points s'(x) vs. x with a 10:8 scaling ratio over the same range of image data as in Fig. 2A;

Fig. 2C is a graph (related to image reduction) of reconstructed image data points s'(x) vs. x with a 2:5 scaling ratio over the same range of image data as in Fig. 2A;

Fig. 3 is a block diagram of a first embodiment of a method for scaling an image in two-dimensional form where the scaling ratio may be changed, but is the same in both dimensions;

Fig. 4 is a block diagram of a second embodiment of a method for scaling an image in two dimensional form where the scaling ratio may be changed by different amounts in both dimensions;

Fig. 5 is a schematic block diagram of an apparatus for scaling an image according to the method of Fig. 3;

Fig. 6 is a schematic block diagram of steps for filtering an image in the frequency domain by a method mathematically equivalent to a convolution in the spatial domain;

Fig. 7 illustrates overlapping between adjacent groups of pixels in an image;

Fig. 8 is a schematic block diagram of an apparatus for filtering an image according to the method of Fig. 6; and

Fig. 9 is a schematic block diagram of a method for reducing an image by decimation.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

This invention relates generally to improved methods and apparatus for image processing. More particularly, the invention relates to novel methods and apparatus for using discrete cosine transformations to: enlarge an image by interpolation; reduce an image by decimation; and/or filter an image in the

frequency domain by a method equivalent to a mathematical convolution in the spatial domain.

The following mathematical discussion, in part, sets forth certain fundamentals relating to forward and inverse discrete cosine transforms (DCTs).

A forward DCT is defined as a mathematical process for transforming image data points from the spatial domain to the frequency or, more particularly, DCT domain. Image data points s(i) in one dimensional form may be transformed from the spatial domain to DCT coefficients S(u) for the frequency domain according to equation (1).

$$S(u) = C_u \sqrt{\frac{2}{N}} \sum_{i=0}^{N-1} s(i) \cos\frac{(2i+1)u\pi}{2N} \tag{1}$$

for $0 \le u \le (N-1)$,

where: S(u) represents the DCT coefficients;

s(i) represents the image data points;

N represents the number of image data points;

$C_u = \frac{1}{\sqrt{2}}$ for $u = 0$; and

$C_u = 1$ for $u \neq 0$.

The DCT coefficients S(u) are derived from equation (1) where the cosine normalized basis terms are shown in Fig. 1 for $N = 16$, where $0 \le u \le 15$ and $0 \le i \le 15$. The value for S(0) is determined for $u = 0$ by summing each of the image data points s(i) for $0 \le i \le (N-1)$ times the cosine terms as represented in Fig. 1. The value for S(1) is determined as the summation of image data points s(i) times the cosine terms for $u = 1$. This procedure, which

indexes first on u and then on i, is continued to derive all sixteen DCT coefficients S(0) through S(15).

The various terms of equation (1) can alternatively be expressed in matrix notation, where each cosine term represents an element of a two dimensional matrix defined as a forward DCT basis matrix FB, each image data point represents an element of a first spatial matrix s(i) of image data points, and each DCT coefficient represents an element of a DCT matrix S(u) of DCT coefficients.

An inverse discrete cosine transformation is defined as a mathematical process for transforming DCT coefficients from the DCT domain to reconstructed image data points in the spatial domain. DCT coefficients S(u) in one dimensional form are transformed from the DCT domain to reconstructed image data points s'(i) in the spatial domain according to equation (2).

$$s'(i) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C_u \, S(u) \, \cos\frac{(2i+1)u\pi}{2N} \qquad (2)$$

for $0 \leq i \leq (N-1)$,

where: S(u) represents the DCT coefficients;

s'(i) represents the reconstructed image data points;

N represents the number of DCT coefficients;

$C_u = \frac{1}{\sqrt{2}}$ for u = 0; and

$C_u = 1$ for $u \neq 0$.

If the DCT coefficients S(u) of equation (1) are derived from a set of image data points s(i), and the reconstructed image data points s'(i) of equation (2) are derived from the corresponding DCT coefficients S(u), then $s(i) \equiv s'(i)$ and the

process is referred to as lossless since the reconstructed data points s'(i) are identical to the original data points s(i), within limits. The reconstructed image data points s'(i) are derived from equation (2) where the cosine terms are shown in Fig. 1 for N = 16, where $0 \leq i \leq 15$ and $0 \leq u \leq 15$. The value for s'(0) is determined for $i = 0$ by summing each of the DCT coefficients S(u) times the cosine terms as represented in Fig. 1. The value for s'(1) is determined as the summation of DCT coefficients S(u) times the cosine terms for $i = 1$. This procedure is continued, indexed as before, to derive all sixteen reconstructed image data points s'(0) through s'(15).

Note that the conventional inverse DCT of equation (2) for transforming data in one dimensional form includes the same cosine argument (i.e. same basis functions) used in the conventional forward DCT of equation (1) so that the reconstructed image data points s'(i) coincide with the original image data points s(i). However, there is no recognition in the prior art for one to use the IDCT for determining reconstructed image data points s'(i) that fall between the original image data points s(i) in the spatial domain.

The above examples for a one dimensional DCT and IDCT can be extended, as known by those skilled in the art, to multi-dimensional formats. For instance, Section A.3.3 of ISO/IEC 10918-1 of the draft international standards for digital compression using discrete cosine transforms defines the forward DCT in two dimensional form as:

$$S(v,u) = \frac{1}{4} C_u C_v \sum_{i=0}^{7} \sum_{j=0}^{7} s(j,i) \cos\frac{(2i+1)u\pi}{16} \cos\frac{(2j+1)v\pi}{16} \qquad (3)$$

for $0 \leq u \leq 7$ and $0 \leq v \leq 7$, while defining the IDCT in two dimensional form as:

$$s'(j,i) = \frac{1}{4} \sum_{u=0}^{7} \sum_{v=0}^{7} C_u C_v S(v,u) \cos\frac{(2i+1)u\pi}{16} \cos\frac{(2j+1)v\pi}{16} \quad (4)$$

for $0 \leq i \leq 7$ and $0 \leq j \leq 7$,

where S(v,u) represents DCT coefficients;

s(j,i) represents original image data points;

s'(j,i) represents the reconstructed image data points;

$C_u = \frac{1}{\sqrt{2}}$ for u = 0;

$C_u = 1$ for $u \neq 0$;

$C_v = \frac{1}{\sqrt{2}}$ for v = 0; and

$C_v = 1$ for $v \neq 0$;

According to equation (3), a first spatial matrix s(j,i) (representing a two dimensional 8X8 group of original image data points in the spatial domain) can be forward DCT transformed to an 8X8 DCT matrix S(v,u) in the frequency domain having 64 DCT coefficients that can be related to the 64 image data points through mapping. An 8X8 DCT basis matrix, i.e. forward transform matrix, is derived from the cosine expression of equation (3) by indexing over the full range of values for j and i and v and u.

Once the image data points s(j,i) are transformed into DCT coefficients S(v,u) in the DCT domain, the number of DCT coefficients can be reduced by compression which is defined, generally, as the process of reducing either the bandwidth or the number of bits necessary to represent an image and, more specifically, as the process of decreasing the number of DCT coefficients S(v,u) in the DCT domain by removing a selected set of the DCT coefficients. The selected set are determined to be non-essential to image reproduction according to some predetermined criteria. Typically, the set of DCT coefficients selected

for removal includes zero or near zero values or those terms which represent high frequency content that the human visual system cannot perceive and therefore suffers no information loss.

Another aspect of image processing in the DCT domain is sharpening and smoothing an image by a filtering procedure mathematically equivalent to a convolution in the spatial domain. A convolution of two discrete signals in the spatial domain occurs by multiplying the two discrete signals point-by-point then summing the products over appropriate limits. Sharpening is defined as the process of enhancing blurry images, particularly by emphasizing high frequency components representing edges in an image. Smoothing, on the other hand, is defined as the process for either softening the edges of the image or alternatively decreasing high frequency components.

Filtering an image in the DCT domain using a process similar to a mathematical convolution in the spatial domain is disclosed in an article entitled "Discrete Cosine Transform Filtering" by Chitprasert & Rao, Signal Processing 19(1990) pgs. 233-245, where a kernel is processed through a 2N point discrete Fourier transform (DFT) yielding 2N complex numbers. A kernel is defined as the signal values of a filter to perform a specific operation such as sharpening or smoothing in the spatial or frequency domain. The imaginary parts of the 2N complex numbers are determined, then discarded, while the real and even parts of the 2N complex numbers are retained for further calculations. Determination of the imaginary parts of the complex numbers requires computational time and effort even though the imaginary parts are not required for filtering the image. However, this filtering process is computationally intensive and, as will be seen, its requirement for use of complex numbers will be overcome by the invention.

Scaling By Resampling

According to the invention, images may be scaled utilizing hybrid procedures based on, but advantageously different from, the foregoing mathematical description.

An image can be scaled in accordance with a scaling ratio defined for each dimension as the number of output pixels desired after a transformation divided by the number of input pixels available before the transformation. For example, for one dimensional image data where the number of input pixels N is eight pixels per cm and the number of output pixels N' is ten pixels per cm, the scaling ratio $R = N'/N$ is 10:8. To enlarge an image then, a scaling ratio greater than one is selected (e.g., 10:8). To reduce an image, a scaling ratio less than one is selected (e.g., 6:8) and when a scaling ratio of one is selected (e.g, 8:8), the output image is the same size as the input image.

Examples of scaling one dimensional image data are shown in Figs. 2A, 2B and 2C. Fig 2A is a plot of the values from a second spatial matrix, $s'(i)$, image data points in one-dimension (designated by open circles), which were derived from equation (2) for the inverse transformation of a DCT matrix $S(u)$ of eight DCT coefficients for $0 \leq i \leq 7$. The IDCT of equation (2) for one dimensional data, nor equation (4) for two dimensional data, does not reconstruct image data points which fall between the discrete index values for the original image data points. However, there are instances in image processing when calculation of such intermediate values is desirable, e.g., image enlargement by interpolation and image reduction by decimation.

A reconstructed matrix of image data points $s'(x)$ for one dimensional data can be derived from a hybrid IDCT which is given by equation (5). In other words, the hybrid IDCT of equation (5) provides for a continuous index variable, x, for the range of $i(N-1)$, so that the reconstructed image data points $s'(x)$ may be determined at non-integer values of x (e.g., $x_1 = 0.5$, $x_2 = 6.3$, $x_3 = 4.99712$, etc.). Instead of indexing i with an interval of delta i equal to the interger value of 1 (the sampling rate for the original image data), x can be indexed at some other sampling rate, Δx, which can be smaller or larger than one in floating point form, as the case may be.

Due to the scaling, some of the reconstructed image data will correspond to spatial locations different than the original image data points of the first spatial

matrix s(j,i). In this manner, reconstructed image data points s'(x) can be determined and these will fall between the discrete index values of the original image data points s(i).

$$s'(x) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C_u \; S(u) \; \cos\frac{(2x+1)u\pi}{2N} \tag{5}$$

for $0 \leq x \leq (N-1)$,

where: s'(x) represents reconstructed image data points;

S(u) represents the DCT coefficients;

N represents the number of DCT coefficients;

x is a continuous variable over the range $0 \leq x \leq N-1$;

$C_u = \frac{1}{\sqrt{2}}$ for u = 0; and

$C_u = 1$ for $u \neq 0$.

An infinite number of reconstructed image data points, s'(x), can be theoretically derived to construct, for example, the curves of Figs. 2B and 2C, while keeping within the appropriate range of the function s'(x) of equation (5). The index, x, of the resampled image data points (as shown in Figs. 2B and 2C) is determined by first selecting a scaling ratio, R. The scaling ratio is selected for each dimension of image data, e.g., one dimensional image data is scaled by a single scaling ratio, two dimensional image data is scaled in the first dimension by a first dimension scaling ratio and in the second dimension by a second dimension scaling ratio, etc., which may be the same or different.

The end points of contiguous groups of image data points (i.e., end points of each row and column of the image data points in matrix notation) are sometimes omitted during scaling but can be retained or recaptured if desired.

Fig. 2B shows an example of scaling up by resampling (or upsampling) in one dimension, where both end points have arbitrarily been retained. The spacing of the x index values in Fig. 2B is determined for a scaling ratio of 10:8, meaning ten reconstructed image data points s'(x) are sought for each eight image data points s(i) of the original image. In other words, the source axis, i.e. the i axis of Fig. 2A, is divided into seven segments (for eight image data points) and the target axis, i.e. the x axis of Fig. 2B, is divided into nine segments (for ten image data points). Hence, s'(x) for $0 \leq x \leq N$ is solved according to the relationship $1/9 = x/7$, where $x = 7/9$. In other words the ten image data points s'(x) of Fig. 2B, derived according to the 10:8 scaling ratio, are resampled for s(x) at 0, 7/9, 14/9, 21/9, 28/9, 35/9, 42/9, 49/9, 56/9, and 7 with respect to the original i-axis. In general, the new sampling interval for interpolation including both end points is given by: $\Delta x = \dfrac{N'-1}{N-1}$

Fig. 2C shows an example of scaling down by resampling (or downsampling) in one dimension where a 2:5 scaling ratio is chosen and the zero index point (i.e. end point) is ignored. The scaling ratio of 2:5 indicates that only 0.4 of the number of originally sampled points are to be resampled. Solving the spacing between resampled image data points is determined as 2.5. Hence, the resampled points designated by the values for x fall within the given range of $0 \leq x \leq 7$ (omitting the zero end point) for s'(x) equals 2.5 and 5.0 as shown in Fig. 2C. The pitch between adjacent pixels is the inverse of the sampling ratio.

<u>Enlarging An Image By Interpolation</u>

One special case of scaling by resampling is enlarging an image by interpolation, defined as the process of generating (by approximation) a greater number N' of reconstructed image data points s'(x) than the original number N of image data points s(i).

Figure 3 is a block diagram of a preferred embodiment of a method of scaling by resampling an image of two dimensional data where the first dimension scaling ratio and the second dimension scaling ratio are equal and do not equal one. The forward DCT section is denoted by reference number 10 whereas the hybrid IDCT section is denoted by the reference number 12. A first spatial matrix s(j,i), as in the forward DCT equation (3), is selected in block 20 as an 8X8 group of pixels in the spatial domain example (shown below). Of course, it will be understood that the 8X8 matrix is usually but a subset of a larger set of original image data which has been remapped into smaller submatrices that serve as the input to the inventive methods and/or apparatus. In JPEG format, the adopted standard submatrix size may be 8x8 or 16x16. However, the pixel group, in the most general sense, can be of any convenient size that can be selected as a program parameter or directly by an operator.

FIRST SPATIAL MATRIX, s(j,i)

100.00	120.00	140.00	160.00	100.00	100.00	100.00	100.00
100.00	100.00	120.00	140.00	160.00	100.00	100.00	100.00
120.00	100.00	100.00	120.00	140.00	160.00	100.00	100.00
140.00	120.00	100.00	100.00	120.00	140.00	160.00	100.00
160.00	140.00	120.00	100.00	100.00	120.00	140.00	160.00
100.00	120.00	140.00	160.00	100.00	100.00	100.00	100.00
100.00	100.00	120.00	140.00	160.00	100.00	100.00	100.00
120.00	100.00	100.00	120.00	140.00	160.00	100.00	100.00

Each element of the first spatial matrix s(j,i) represents an image data point corresponding to a pixel of an image. The pixel data is thus stored in the spatial domain in matrix form.

The following 8X8 forward DCT basis matrix FB is produced in block 22 by evaluating the cosine terms of equation (3) for the two dimensional forward DCT by indexing first on u and v and then on i and j.

FORWARD DCT BASIS MATRIX, FB

0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.354
0.490	0.416	0.278	0.098	-0.098	-0.278	-0.416	-0.490
0.462	0.191	-0.191	-0.462	-0.462	-0.191	0.191	0.462
0.416	-0.098	-0.490	-0.278	0.278	0.490	0.098	-0.416
0.354	-0.354	-0.354	0.354	0.354	-0.354	-0.354	0.354
0.278	-0.490	0.098	0.416	-0.416	-0.098	0.490	-0.278
0.191	-0.462	0.462	-0.191	-0.191	0.462	-0.462	0.191
0.098	-0.278	0.416	-0.490	0.490	-0.416	0.278	-0.098

A first intermediate matrix I_1 as shown below is an 8X8 matrix generated in multiplier 25 by matrix product multiplication of the 8X8 forward DCT basis matrix FB times the 8X8 first spatial matrix s(j,i) (i.e., I_1 = FB * s(j,i)).

FIRST INTERMEDIATE MATRIX, I_1

332.340	318.198	332.340	367.696	360.624	346.482	318.198	304.056
-6.203	2.301	6.553	8.504	-6.553	-10.806	1.951	-5.853
-40.782	-22.304	9.239	36.955	24.546	-11.480	-46.194	-27.716
-12.567	23.678	41.801	36.245	-41.801	-59.923	-5.556	16.667
35.355	21.213	-7.071	-28.284	-35.355	21.213	35.355	21.213
-11.920	-4.710	-1.105	7.210	1.105	-2.500	8.315	-24.944
-6.069	1.585	3.827	15.307	-33.128	27.716	-19.134	-11.480
16.172	3.444	-2.920	-12.728	2.920	9.283	-9.808	29.424

In block 24, the following 8X8 first transpose matrix, FB^T, is generated by transposing the above forward DCT basis matrix, FB.

FIRST TRANSPOSE MATRIX, FB^T

0.354	0.490	0.462	0.416	0.354	0.278	0.191	0.098
0.354	0.416	0.191	-0.098	-0.354	-0.490	-0.462	-0.278
0.354	0.278	-0.191	-0.490	-0.354	0.098	0.462	0.416
0.354	0.098	-0.462	-0.278	0.354	0.416	-0.191	-0.490
0.354	-0.098	-0.462	0.278	0.354	-0.416	-0.191	0.490
0.354	-0.278	-0.191	0.490	-0.354	-0.098	0.462	-0.416
0.354	-0.416	0.191	0.098	-0.354	0.490	-0.462	0.278
0.354	-0.490	0.462	-0.416	0.354	-0.278	0.191	-0.098

The 8X8 first intermediate matrix I_1 is next matrix product multiplied in multiplier 26 times the 8X8 first transpose matrix, FB^T, to generate the following 8X8 DCT matrix S(v,u) of DCT coefficients as defined in equation (3) (i.e. $S(v,u) = I_1 * FB^T$).

DCT MATRIX, S(v,u)

947.500	10.632	-50.581	16.730	17.500	9.417	2.010	-6.588
-3.573	6.265	-4.483	-12.875	-3.573	7.684	-6.609	-0.299
-27.484	10.490	-72.730	-21.370	22.537	-8.165	5.732	-5.382
-0.515	33.688	11.396	-86.570	-0.515	19.912	-14.895	-6.955
22.500	-6.111	63.647	19.165	-27.500	11.044	3.402	-9.918
-10.094	1.955	-19.490	4.306	-10.094	12.679	-11.975	2.475
-7.557	9.356	-9.268	-1.511	-17.453	9.149	22.730	-38.912
12.653	-5.906	23.158	3.529	12.653	17.875	16.480	-2.374

Once the DCT matrix S(v,u) is determined as shown above, equation (4) is typically used to generate a second spatial matrix s'(j,i) of reconstructed image data points derived from the inverse discrete cosine transformation of DCT matrix S(v,u).

The inventive method of Fig. 3 allows resampling to generate an infinite number of reconstructed image data points s'(y,x) in two dimensional form over the given ranges of the hybrid IDCT of equation (6). The values of real number indices x and y are determined according to the scaling ratios previously discussed for the one dimensional IDCT example of Fig. 2B.

$$s'(y,x) = \sqrt{\frac{2}{N}}\sqrt{\frac{2}{M}}\sum_{u=0}^{N-1}\sum_{v=0}^{M-1} S(v,u) \cos\frac{(2x+1)u\pi}{2N}\cos\frac{(2y+1)v\pi}{2M} \tag{6}$$

for $0 \leq x \leq (N - 1)$ and $0 \leq y \leq (M - 1)$,

where $s'(y,x)$ is a two dimensional matrix of reconstructed image data points;

$S(v,u)$ represents a two dimensional matrix of DCT coefficients;

N represents the number of DCT coefficients in the first dimension; and

M represents the number of DCT coefficients in the second dimension.

Since the first dimension scaling ratio equals the second dimension scaling ratio for the preferred embodiment of Fig. 3, then $N'=M'$ (output pixels in the second dimension) for equation (6).

The conventional size of a two dimensional IDCT basis matrix is 8X8 in conformance with the above JPEG standard of equation (4). To enlarge each pixel group size by 25%, the first dimension scaling ratio and the second dimension scaling ratio are each selected in block 23 as 10:8. In other words, $N' = M' = 10$, which means that the first dimension hybrid IDCT basis matrix IB_{1HYB} generated in block 30 will be a 10X8 matrix as shown below.

FIRST DIMENSION HYBRID IDCT BASIS MATRIX, IB_{1HYB}

0.354	0.490	0.462	0.416	0.354	0.278	0.191	0.098
0.354	0.438	0.269	0.033	-0.211	-0.403	-0.496	-0.466
0.354	0.346	-0.022	-0.376	-0.498	-0.313	0.065	0.403
0.354	0.221	-0.304	-0.490	-0.129	0.376	0.462	0.033
0.354	0.076	-0.477	-0.221	0.410	0.346	-0.304	-0.438
0.354	-0.076	-0.477	0.221	0.410	-0.346	-0.304	0.438
0.354	-0.221	-0.304	0.490	-0.129	-0.376	0.462	-0.033
0.354	-0.346	-0.022	0.376	-0.498	0.313	0.065	-0.403
0.354	-0.438	0.269	-0.033	-0.211	0.403	-0.496	0.466
0.354	-0.490	0.462	-0.416	0.354	-0.278	0.191	-0.098

In multiplier 32, a 10X8 second intermediate matrix I_2 (shown below) is generated by matrix product multiplication of the 10X8 first dimension hybrid IDCT basis matrix IB_{1HYB} times the 8X8 DCT matrix S(v,u), i.e. $I_2 = IB_{1HYB} * S(v,u)$.

SECOND INTERMEDIATE MATRIX, I_2

325.269	25.279	-31.543	-38.234	0.000	19.038	-2.242	-18.350
323.191	9.042	-50.960	-14.983	23.295	1.504	-15.971	17.360
331.110	-6.306	-39.125	24.903	25.473	-17.401	14.082	0.969
333.036	-8.914	-21.492	50.581	-9.094	4.728	10.906	-13.185
350.419	-10.303	25.909	42.521	-19.705	17.362	-16.121	11.431
368.808	-2.886	65.467	6.308	-1.316	0.561	1.025	4.607
340.873	20.272	4.966	-32.098	-1.258	12.495	7.145	-21.580
316.672	20.676	-58.083	-31.431	11.035	14.608	-13.333	-0.591
330.009	-2.581	-41.594	8.729	30.114	-12.970	-3.501	17.860
332.340	-8.810	-29.958	43.294	7.071	-8.612	20.063	-13.185

The first dimension hybrid IDCT basis matrix IB_{1HYB} is transposed in block 28 to generate the following 8X10 second transpose matrix, IB_{1HYB}^T.

SECOND TRANSPOSE MATRIX, IB_{1HYB}^T

0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.354
0.490	0.438	0.346	0.221	0.076	-0.076	-0.221	-0.346	-0.438	-0.490
0.462	0.269	-0.022	-0.304	-0.477	-0.477	-0.304	-0.022	0.269	0.462
0.416	0.033	-0.376	-0.490	-0.221	0.221	0.490	0.376	-0.033	-0.416
0.354	-0.211	-0.498	-0.129	0.410	0.410	-0.129	-0.498	-0.211	0.354
0.278	-0.403	-0.313	0.376	0.346	-0.346	-0.376	0.313	0.403	-0.278
0.191	-0.496	0.065	0.462	-0.304	-0.304	0.462	0.065	-0.496	0.191
0.098	-0.466	0.403	0.033	-0.438	0.438	-0.033	-0.403	0.466	-0.098

Finally in multiplier 33, the 10X10 second spatial matrix s'(y,x) of equation (6), which represents the reconstructed image data points of the 8X8 first spatial matrix s(j,i), is determined as follows by matrix product multiplication of the 10X8 second intermediate matrix I_2 times the 8X10 second transposed matrix IB_{1HYB}^T (i.e., $s'(y,x) = I_2 * IB_{1HYB}^T$).

SECOND SPATIAL MATRIX, s'(y,x)

100.00	118.34	125.30	154.46	155.73	105.72	92.67	105.79	96.93	100.00
96.22	98.35	118.02	129.86	149.88	156.06	108.90	87.44	108.79	95.59
113.21	98.81	100.44	112.07	129.44	154.30	152.30	111.85	89.58	108.17
123.37	110.47	94.56	105.07	116.50	125.40	155.93	152.35	106.50	90.01
144.37	127.55	111.72	94.88	99.18	117.57	127.34	152.46	158.46	107.25
162.18	144.32	128.00	107.74	94.88	101.76	114.48	131.37	151.18	158.55
121.70	131.43	127.96	146.68	137.88	93.04	98.25	115.04	105.72	125.77
87.55	103.05	121.01	147.51	162.07	134.40	96.59	92.71	98.21	85.41
107.94	96.93	109.44	114.68	135.47	164.35	132.97	95.27	104.81	106.94
120.00	105.18	94.00	108.12	121.13	136.02	161.82	137.88	90.84	100.00

The second spatial matrix s'(y,x) provides reconstructed image data points which are used for reproduction of the image in block 36. Note that the original image of the above two dimensional interpolation example was represented by an 8X8 pixel group which was increased to a 10X10 pixel group by resampling using interpolation, according to the first dimension hybrid IDCT basis matrix IB_{1HYB}

derived from equation (6) as described above. This method facilitates image enlargement with excellent fidelity without compromising resolution.

The above preferred embodiment of the method of Fig. 3 is appropriate for two dimensional data when: the first dimension scaling ratio equals one and the second dimension scaling ratio does not equal one; the first dimension scaling ratio does not equal one and the second dimension scaling ratio equals one; and the first dimension scaling ratio does not equal one, the second dimension scaling ratio does not equal one, and the first dimension scaling ratio equals the second dimension scaling ratio. When different scaling ratios are selected by independent resampling of the image data points for each dimension, the method of Fig. 3 is modified as reflected in Fig. 4.

Fig. 4 is a block diagram, including a forward DCT section 10 and a hybrid IDCT section 12, of a second embodiment of a method for scaling an image in two dimensional form where the first dimension scaling ratio does not equal one, the second dimension scaling ratio does not equal one, and the first dimension scaling ratio does not equal the second dimension scaling ratio. For example, for an 8X8 first spatial matrix s(j,i), the first dimension scaling ratio is selected in block 23 as 10:8 (N' = 10) whereas the second dimension scaling ratio is selected as 12:8 (M' = 12). In this case, the 10X8 first dimension hybrid IDCT basis matrix IB_{1HYB} is generated in block 30, and the 8X12 second dimension hybrid IDCT basis matrix IB_{2HYB} is generated in block 38. Multiplier 44 produces the 10X8 second intermediate matrix I_2 by matrix product multiplication of the 10X8 first dimension hybrid IDCT basis matrix IB_{1HYB} times the 8X8 DCT matrix S(v,u). In multiplier 46, the second spatial matrix s'(j,i) is generated by matrix product multiplication of the 10X8 second intermediate matrix I_2 times the 8X12 second dimension hybrid IDCT basis matrix IB_{2HYB} (i.e. $s'(y,x) = I_2 * IB_{2HYB}$). From the second spatial matrix s'(j,i), the image can be printed, processed, or otherwise reconstructed in block 36.

An apparatus corresponding to the method of Fig. 3 is shown in Fig. 5. A forward discrete cosine transform processor 50 transforms the first spatial matrix s(j,i) into DCT matrix S(v,u) in matrix multiplier 54. After processing the DCT matrix S(v,u) in the frequency domain for instance, for compression (in a processor not shown), the second spatial matrix s'(y,x) of reconstructed image data points is generated in matrix multiplier 66 of the hybrid IDCT processor 62.

A digital representation of an image (not shown) is represented by the first spatial matrix s(j,i) and input to the forward DCT processor 50, which includes a cosine processor 52, a transposer 51 and a matrix multiplier 54. The cosine processor 52 generates the forward DCT basis matrix FB in the DCT domain by processing the cosine terms of equation (3). The transposer 51 produces the first transpose matrix FB^T by transposing the forward DCT basis matrix FB. The matrix multiplier 54 initially produces the first intermediate matrix I_1 by matrix product multiplication of the forward DCT basis matrix FB times the first spatial matrix s(j,i). The matrix multiplier 54 then generates the DCT matrix S(v,u) of DCT coefficients in the frequency domain by matrix product multiplication of the first intermediate matrix I_1 times the first transpose matrix FB^T.

If desired, the DCT matrix S(v,u) can be decreased in size (i.e. decimated) in a microprocessor or other appropriate circuitry (not shown). Interpolation is typically provided in the spatial domain in the form of linear interpolation. Interpolation or increasing the number of reconstructed image data points s'(y,x) can be accomplished in the hybrid IDCT processor 62, which includes a resampling cosine processor 64, a transposer 76 and a matrix multiplier 66. The resampling cosine processor 64 produces a first dimension hybrid IDCT basis matrix IB_{1HYB} in response to the first dimension scaling ratio received at terminal 68 along with the second dimension scaling ratio. Note that the first dimension scaling ratio does not equal one, the second dimension scaling ratio does not equal one, and the first dimension scaling ratio equals the second dimension scaling ratio. The elements of the first dimension hybrid IDCT basis

matrix IB_{1HYB} are derived from the first dimension cosine terms of equation (6). The transposer 76 produces the second transposed matrix IB_{1HYB}^T by transposing the first dimension hybrid IDCT basis matrix IB_{1HYB}. The second intermediate matrix I_2 is generated by matrix product multiplication of the first dimension hybrid IDCT basis matrix IB_{1HYB} matrix times the DCT matrix S(v,u) in matrix multiplier 66. Finally, the matrix multiplier 66 generates the second spatial matrix s'(y,x) by matrix product multiplication of the second intermediate matrix I_2 times the second transpose matrix IB_{1HYB}^T. The image can now be printed or otherwise reconstructed using the reconstructed image data of s'(y,x).

The above methods of Fig. 3 and 4 for enlarging an image by interpolation can also be used for reducing an image by decimation using downsampling, where the scaling ratios are selected to be less than one, and the image data is well behaved for this application. However for best results, the method of reducing an image by decimation using downsampling is generally preceded by low pass filtering to remove high frequency components which might cause aliasing.

Filtering

Typically, filtering an image in the DCT domain in a manner similar to a mathematical convolution in the spatial domain involves operations with a matrix of DCT coefficients derived from a matrix of kernel values represented as a DCT kernel matrix of DCT kernel coefficients.

Convolution of an image, as understood by those of skill in the art, is a process of filtering in the spatial domain by multiplying two discrete signals point-by-point then summing the products over appropriate limits. Convolution generally results in filtering by sharpening or smoothing an image. Sharpening enhances blurry images, particularly by enhancing high frequency components representing edges in an image, whereas smoothing softens the edges of the image. The kernel is defined as the signal values of a filter used for performing a specific filtering operation such as sharpening or smoothing in the spatial or

frequency domain. The kernel may be selected in the spatial or frequency domain according to a designer's predetermined criteria for sharpening, smoothing, or the like.

Known methods for filtering an image in the frequency domain, in a manner similar to a mathematical convolution in the spatial domain, require computation of complex numbers when processing the kernel as previously described. The following filtering method performed in the DCT domain obviates the need for the calculation of imaginary numbers when processing the kernel, thus minimizing computational complexity, time and effort.

Image data is filtered in the DCT domain in a manner mathematically equivalent to a convolution in the spatial domain for one dimensional data by: (1) generating a DCT matrix S(u) of DCT coefficients by taking a discrete even cosine transform (DECT) of a first spatial matrix s(i) of image data points; (2) generating a DOCT matrix H(u) of DOCT coefficients by taking a discrete odd cosine transform (DOCT) of a kernel matrix h(i); (3) generating a mask multiplied matrix by mask multiplying S(u) with H(u); and (4) generating a second spatial matrix s'(i) of reconstructed image data points by taking an IDCT of the mask multiplied matrix. It will be understood by those skilled in the art that a discrete even cosine transformation (DECT) is conventionally referred to as a discrete cosine transformation (DCT). Also, in a preferred embodiment the kernel h(i) is odd and symmetric.

Since images contain varying amounts of high frequency content, aliasing often results during transformation operations. When the amount of high frequency content is problematic, then the above filtering procedure would further include the steps of: selecting an overlap of adjacent first spatial matrices s(i) prior to generating DCT matrices S(u); and generating filtered saved regions $s'_S(i)$ of the second spatial matrices s'(i) by discarding certain selected elements of the second spatial matrices s'(i).

Mathematically, the one dimensional expression for filtering in a manner equivalent to a mathematical convolution is defined by equation (7).

$$s(i) \otimes h(i) = S(u) \blacksquare H(u) \tag{7}$$

where s(i) represents the first spatial matrix of image data points;

h(i) represents the kernel matrix in the spatial domain;

s(i) ⊗ h(i) represents the filtering or convolution of s(i) with h(i);

H(u) represents the DOCT matrix derived from h(i);

S(u) represents the DCT matrix derived from s(i);

i and u are integer indices;

⊗ is the convolution operator; and

▪ designates mask multiplication.

The above DCT matrix S(u) for one dimensional data is derived from equation (1), whereas the one dimensional DOCT matrix H(u) is derived as follows from equation (8) where h(i) is an odd symmetric kernel of size k.

$$H(u) = 2\sum_{i=0}^{N-1} d_i \; h(i) \; \cos\frac{iu\pi}{N} \tag{8}$$

for $0 \leq u \leq (N-1)$;

where N represents the size of the DCT block used;

$h(i) = 0$ for $|i| > (k-1)/2$;

$d_i = ½$ for $i=0$;

$d_i = 1$ for $i=1, 2 \ldots (N-1)$;

i, u and N are integers; and

k is the kernel size.

Of course, the above mathematical filtering analysis could be readily extended to image data presented in any number of dimensions.

A preferred embodiment of a method of filtering two dimensional image data in the DCT domain, in a manner equivalent to a mathematical convolution in the spatial domain, is depicted in Fig. 6 which includes a forward DCT section 10 having steps identical to those previously described for the interpolation method of Fig. 3. As an example, the 8X8 first spatial matrix s(j,i) of 64 image data points listed below is selected in block 20 as shown below.

FIRST SPATIAL MATRIX, s(j,i)

100.00	120.00	140.00	160.00	100.00	100.00	100.00	100.00
100.00	100.00	120.00	140.00	160.00	100.00	100.00	100.00
120.00	100.00	100.00	120.00	140.00	160.00	100.00	100.00
140.00	120.00	100.00	100.00	120.00	140.00	160.00	100.00
160.00	140.00	120.00	100.00	100.00	120.00	140.00	160.00
100.00	120.00	140.00	160.00	100.00	100.00	100.00	100.00
100.00	100.00	120.00	140.00	160.00	100.00	100.00	100.00
120.00	100.00	100.00	120.00	140.00	160.00	100.00	100.00

The following 8X8 forward DECT basis matrix FB_E is derived from the cosine terms of equation (3) in block 22.

FORWARD DECT BASIS MATRIX, FB_E

0.354	0.354	0.354	0.354	0.354	0.354	0.354	0.354
0.490	0.416	0.278	0.098	-0.098	-0.278	-0.416	-0.490
0.462	0.191	-0.191	-0.462	-0.462	-0.191	0.191	0.462
0.416	-0.098	-0.490	-0.278	0.278	0.490	0.098	-0.416
0.354	-0.354	-0.354	0.354	0.354	-0.354	-0.354	0.354
0.278	-0.490	0.098	0.416	-0.416	-0.098	0.490	-0.278
0.191	-0.462	0.462	-0.191	-0.191	0.462	-0.462	0.191
0.098	-0.278	0.416	-0.490	0.490	-0.416	0.278	-0.098

The following 8X8 first intermediate matrix I_1 is generated in multiplier 25 by matrix product multiplication of the 8X8 forward DECT matrix FB_E times the 8X8 first spatial matrix s(j,i).

FIRST INTERMEDIATE MATRIX, I_1

332.340	318.198	332.340	367.696	360.624	346.482	318.198	304.056
-6.203	2.301	6.553	8.504	-6.553	-10.806	1.951	-5.853
-40.782	-22.304	9.239	36.955	24.546	-11.480	-46.194	-27.716
-12.567	23.678	41.801	36.245	-41.801	-59.923	-5.556	16.667
35.355	21.213	-7.071	-28.284	-35.355	21.213	35.355	21.213
-11.920	-4.710	-1.105	7.210	1.105	-2.500	8.315	-24.944
-6.069	1.585	3.827	15.307	-33.128	27.716	-19.134	-11.480
16.172	3.444	-2.920	-12.728	2.920	9.283	-9.808	29.424

In block 24, the following 8X8 first transpose matrix FB_E^T is generated by transposing the above 8X8 forward DCT basis matrix FB_E.

FIRST TRANSPOSE MATRIX, FB_E^T

0.354	0.490	0.462	0.416	0.354	0.278	0.191	0.098
0.354	0.416	0.191	-0.098	-0.354	-0.490	-0.462	-0.278
0.354	0.278	-0.191	-0.490	-0.354	0.098	0.462	0.416
0.354	0.098	-0.462	-0.278	0.354	0.416	-0.191	-0.490
0.354	-0.098	-0.462	0.278	0.354	-0.416	-0.191	0.490
0.354	-0.278	-0.191	0.490	-0.354	-0.098	0.462	-0.416
0.354	-0.416	0.191	0.098	-0.354	0.490	-0.462	0.278
0.354	-0.490	0.462	-0.416	0.354	-0.278	0.191	-0.098

The first intermediate matrix I_1 is matrix product multiplied in multiplier 26 times the first transpose matrix FB_E^T to generate the following DCT matrix S(v,u) of DCT coefficients as defined in equation (3) (i.e. $S(v,u) = I_1 * FB_E^T$).

DCT MATRIX, S(v,u)

947.500	10.632	-50.581	16.730	17.500	9.417	2.010	-6.588
-3.573	6.265	-4.483	-12.875	-3.573	7.684	-6.609	-0.299
-27.484	10.490	-72.730	-21.370	22.537	-8.165	5.732	-5.382
-0.515	33.688	11.396	-86.570	-0.515	19.912	-14.895	-6.955
22.500	-6.111	63.647	19.165	-27.500	11.044	3.402	-9.918
-10.094	1.955	-19.490	4.306	-10.094	12.679	-11.975	2.475
-7.557	9.356	-9.268	-1.511	-17.453	9.149	22.730	-38.912
12.653	-5.906	23.158	3.529	12.653	17.875	16.480	-2.374

The above described steps complete the determination of the two dimensional forward DECT matrix S(v,u) necessary in equation (7), Next, the two dimensional forward DOCT matrix H(v,u) must be determined. The following 5X5 odd, symmetrical kernel h(j,i) in the spatial domain was arbitrarily selected in block 124 to illustrate the preferred embodiment of the filtering process.

KERNEL MATRIX, h(j,i)

0.006	0.025	0.037	0.025	0.006
0.025	-0.330	-0.709	-0.330	0.025
0.037	-0.709	4.782	-0.709	0.037
0.025	-0.330	-0.709	-0.330	0.025
0.006	0.025	0.037	0.025	0.006

In order to facilitate multiplication of the 5X5 kernel matrix h(j,i) with other 8X8 matrixes (selected for this example and in conformance with the international JPEG standard matrix size), the 5X5 kernel matrix h(j,i) is padded in block 122 to generate an 8X8 padded kernel matrix $h_p(j,i)$ in the spatial domain. The padding is accomplished by inserting the lower right quadrant of the odd, symmetric kernel into the upper left quadrant of the padded kernel matrix $h_p(j,i)$, then setting the remaining elements of $h_p(j,i)$ to zero as shown below.

PADDED KERNEL MATRIX, $h_p(j,i)$

4.782	-0.709	0.037	0.000	0.000	0.000	0.000	0.000
-0.709	-0.330	0.025	0.000	0.000	0.000	0.000	0.000
0.037	0.025	0.006	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The two dimensional DOCT matrix H(v,u) is derived as follows from equation (9).

$$H(v,u) = 4 \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} d_i \, d_j \, h_P \, (j,i) \, \cos\frac{iu\pi}{N} \, \cos\frac{jv\pi}{M} \qquad (9)$$

for $0 \leq u \leq N\text{-}1$ and $0 \leq v \leq N\text{-}1$,

where $h_P(j,i)$ is the two dimensional padded kernel matrix;

N is the number of elements of $h_P(j,i)$ in the first dimension;

M is the number of elements of $h_P(j,i)$ in the second dimension;

$d_i = ½$ for $i = 0$;

$d_i = 1$ for $i = 1, 2, \ldots (N\text{-}1)$;

$d_j = ½$ for $j = 0$;

$d_j = 1$ for $j = 1, 2, \ldots (M\text{-}1)$;

i, j, u, v, N, M are integers; and

$k_p(j,i) = 0$ for $|\,i\,|$ or $|\,j\,| > \frac{(k-1)}{2}$

An 8X8 forward DOCT basis matrix FB_O (listed below) is derived from the cosine terms of equation (9).

FORWARD DOCT BASIS MATRIX, FB_O

1.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
1.000	1.848	1.414	0.765	-0.000	-0.765	-1.414	-1.848
1.000	1.414	-0.000	-1.414	-2.000	-1.414	0.000	1.414
1.000	0.765	-1.414	-1.848	0.000	1.848	1.414	-0.765
1.000	-0.000	-2.000	0.000	2.000	-0.000	-2.000	0.000
1.000	-0.765	-1.414	1.848	-0.000	-1.848	1.414	0.765
1.000	-1.414	0.000	1.414	-2.000	1.414	-0.000	-1.414
1.000	-1.848	1.414	-0.765	0.000	0.765	-1.414	1.848

In multiplier 128, the following 8X8 second intermediate matrix I_2 is generated by matrix product multiplication of the 8X8 forward DOCT basis matrix FB_O times the 8X8 padded kernel matrix $h_P(j,i)$.

SECOND INTERMEDIATE MATRIX, I_2

3.439	-1.319	0.099	0.000	0.000	0.000	0.000	0.000
3.525	-1.283	0.092	0.000	0.000	0.000	0.000	0.000
3.779	-1.175	0.072	0.000	0.000	0.000	0.000	0.000
4.187	-0.996	0.047	0.000	0.000	0.000	0.000	0.000
4.708	-0.759	0.025	0.000	0.000	0.000	0.000	0.000
5.272	-0.492	0.009	0.000	0.000	0.000	0.000	0.000
5.785	-0.243	0.002	0.000	0.000	0.000	0.000	0.000
6.145	-0.065	0.000	0.000	0.000	0.000	0.000	0.000

The 8X8 forward DOCT basis matrix FB_O is transposed in block 130 to produce the following 8X8 second transposed matrix, $FB_O{}^T$.

SECOND TRANSPOSED MATRIX, $FB_O{}^T$

1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.000	1.848	1.414	0.765	-0.000	-0.765	-1.414	-1.848
2.000	1.414	-0.000	-1.414	-2.000	-1.414	0.000	1.414
2.000	0.765	-1.414	-1.848	0.000	1.848	1.414	-0.765
2.000	-0.000	-2.000	0.000	2.000	-0.000	-2.000	0.000
2.000	-0.765	-1.414	1.848	-0.000	-1.848	1.414	0.765
2.000	-1.414	0.000	1.414	-2.000	1.414	-0.000	-1.414
2.000	-1.848	1.414	-0.765	0.000	0.765	-1.414	1.848

Multiplier 136 generates the following 8X8 forward DOCT matrix H(v,u) by matrix product multiplication of the 8X8 second intermediate matrix I_2 times the second transpose matrix FB_O.

FORWARD DOCT MATRIX, H(v,u)

1.000	1.143	1.574	2.289	3.240	4.307	5.303	6.016
1.143	1.284	1.710	2.413	3.341	4.377	5.339	6.025
1.574	1.710	2.117	2.778	3.635	4.576	5.441	6.053
2.289	2.413	2.778	3.357	4.092	4.882	5.596	6.095
3.240	3.341	3.635	4.092	4.658	5.253	5.780	6.144
4.307	4.377	4.576	4.882	5.253	5.635	5.967	6.194
5.303	5.339	5.441	5.596	5.780	5.967	6.128	6.236
6.016	6.025	6.053	6.095	6.144	6.194	6.236	6.265

At this point, both functions S(v,u) and H(v,u) located on the right side of equation (7) have been determined. Thus, by mask multiplying in mask multiplier 144, the 8X8 forward DCT matrix S(v,u) times the 8X8 forward DOCT matrix H(v,u), the following 8X8 third intermediate matrix, I_3, is generated.

THIRD INTERMEDIATE MATRIX, I_3

947.500	12.147	-79.602	38.292	56.698	40.562	10.658	-39.631
-4.082	8.044	-8.283	-31.065	-11.935	33.631	-35.284	-1.801
-43.253	17.941	-153.998	-59.356	81.912	-37.366	31.191	-32.580
-1.178	81.280	31.652	-290.610	-2.105	97.215	-83.353	-42.392
72.897	-20.417	231.332	78.419	-128.091	58.017	19.667	-60.943
-43.477	8.557	-89.196	21.021	-53.022	71.447	-71.460	15.329
-40.080	49.954	-50.429	-8.453	-100.885	54.596	139.289	-252.666
76.113	-35.583	140.178	21.509	77.744	-110.722	102.776	-14.871

To reconstruct the filtered image, the DCT coefficients of the third intermediate matrix I_3 are first transformed to the spatial domain. The 8x8 second spatial matrix s'(j,i) of reconstructed image data points shown below is generated in block 140 by performing an inverse discrete cosine transformation as in equation(4) on the 8x8 third intermediate matrix, I_3. The filtered image can then be printed or otherwise reproduced from the reconstructed image data points of the second spatial matrix s'(v,u).

SECOND SPATIAL MATRIX, s'(j,i)

100.00	118.34	125.30	154.46	155.73	105.72	92.67	105.79	96.93	100.00
96.22	98.35	118.02	129.86	149.88	156.06	108.90	87.44	108.79	95.59
113.21	98.81	100.44	112.07	129.44	154.30	152.30	111.85	89.58	108.17
123.37	110.47	94.56	105.07	116.50	125.40	155.93	152.35	106.50	90.01
144.37	127.55	111.72	94.88	99.18	117.57	127.34	152.46	158.46	107.25
162.18	144.32	128.00	107.74	94.88	101.76	114.48	131.37	151.18	158.55
121.70	131.43	127.96	146.68	137.88	93.04	98.25	115.04	105.72	125.77
87.55	103.05	121.01	147.51	162.07	134.40	96.59	92.71	98.21	85.41
107.94	96.93	109.44	114.68	135.47	164.35	132.97	95.27	104.81	106.94
120.00	105.18	94.00	108.12	121.13	136.02	161.82	137.88	90.84	100.00

As earlier noted, the above method is adequate when the image is relatively free from aliasing due to high frequency components. However, many images require the addition of an overlapping procedure in the above filtering method which is directed at eliminating aliasing due to high frequency components. When the high frequency components do not effectively disturb the desired image quality, then the overlapping procedure can be excluded and the above second spatial matrix s'(j,i) is adequate to print or otherwise reproduce the original image. In the event that aliasing is a problem, then an overlapping procedure such as the one described below in conjunction with Fig. 7 is instituted together with the above method wherein certain reconstructed image data points of the second spatial matrix s'(j,i) are discarded, resulting in a filtered saved region designated as $s'_S(j,i)$.

For a kernel size of 5X5, i.e. k = 5, a k-1 overlap of four pixels between pixel groups is used to provide a 4X4 saved region of filtered pixels for an 8X8 group of pixels to be processed. Thus, for filtering with overlap in a manner similar to a mathematical convolution, each 8X8 pixel group of the image preferably overlaps each adjacent pixel group by four pixels.

Fig. 7 illustrates a preferred embodiment of the overlapping procedure where an image 150 is represented by pixels arranged in twelve rows and twelve columns. A first 8X8 pixel group 152 of 64 image data points spans rows 1-8 and columns 1-8 as designated by the dark, solid line. A second 8X8 pixel

group 154 of 64 image data points spans rows 1-8 and columns 5-12 as designated by a dotted line. Each pixel group represents a two dimensional first spatial matrix s(j,i) for filtering according to the above method of Fig. 8 including overlapping. The overlap region is shown in Fig. 7 as the area including rows 1-8 and columns 5-8. Although the example of Fig. 7 shows overlapping of matrices which are horizontally adjacent in the image 150, the same method applies to matrices which are vertically adjacent in the image 150.

After the first spatial matrix s(j,i) (defined as first pixel group 152) is processed and filtered in the DCT domain, the second spatial matrix s'(j,i) of reconstructed image data points is generated by an IDCT of third intermediate matrix I_3 in block 140 of Fig. 6. Then, in block 142, a perimeter or border area of the second spatial matrix s'(j,i) is determined as (k-1)/2 pixels wide and the reconstructed image data points within the perimeter area are discarded. The remaining pixels constitute the saved matrix s'_S(j,i) of filtered reconstructed pixels. In Fig. 7, all reconstructed image data points for block 152 located in rows and columns 1, 2, 7 and 8 are discarded, resulting in the saved matrix s_S'(j,i) of reconstructed image data points located in rows 3-6, columns 3-6. For second pixel group 154, the second spatial matrix s'(j,i) of reconstructed image data points includes rows 1-8, columns 5-12. The reconstructed filtered pixels discarded in block 142 for second pixel group 154 include all reconstructed pixels in rows 1, 2, 7, 8 and columns 5, 6, 11 and 12. The saved matrix s'_S(j,i) of second pixel group 154 includes reconstructed image data points located at rows 3-6, columns 7-10.

In the case where filtering the image in the frequency domain by a method similar to a mathematical convolution in the spatial domain, is combined with scaling the image, the overlap is determined by taking into consideration the kernel size k.

A filtering system which operates according to the method of Fig. 6 is depicted in the schematic block diagram of Fig. 8. The system includes a forward DECT processor 50, a forward DOCT processor 158, a zero pad

processor 159, a mask multiplier 156, and an IDCT processor 160. The IDCT processor 160 could readily be replaced with a hybrid IDCT processor as previously described in conjunction with Figs. 3 and 4.

DECT processor 50 includes a cosine processor 52, a transposer 51 and a matrix multiplier 54. The first spatial matrix s(j,i) is selected as an 8X8 matrix of image data points and the forward DECT basis matrix FB_E is generated in block 52 as an 8X8 matrix from the cosine terms of equation (3). The first transposed matrix $FB_E{}^T$ is produced as an 8X8 matrix by transposing the forward DECT basis matrix FB_E in transposer 51. The first intermediate matrix I_1 is generated as an 8X8 matrix in matrix multiplier 54 by matrix product multiplication of the forward DECT basis matrix FB_E times the first spatial matrix s(j,i). The forward DECT matrix S(v,u) is then generated in the matrix multiplier 54 as an 8X8 matrix by matrix product multiplication of the 8X8 first intermediate matrix I_1 times the 8X8 first transposed matrix $FB_E{}^T$.

DOCT processor 158 includes a cosine processor 118, a transposer 115, and a matrix multiplier 157. Since the first spatial matrix s(j,i) was selected as an 8X8 matrix of image data points in the forward DECT processor 50, forward DOCT basis matrix FB_O is also generated as an 8X8 matrix in cosine processor 118 from the cosine terms of equation (9). The second transposed matrix $FB_O{}^T$ is produced in transposer 115 as an 8X8 matrix by transposing the forward DOCT basis matrix FB_O. The kernel matrix h(j,i) is input to a zero pad processor 159 which generates the 8X8 padded kernel matrix $h_P(j,i)$. Matrix multiplier 157 generates the second intermediate matrix I_2 as an 8X8 matrix by matrix product multiplication of the forward DOCT basis matrix FB_O times the padded kernel matrix $h_P(j,i)$. The forward DOCT matrix H(v,u) is then generated as an 8X8 matrix in matrix multiplier 157 by matrix product multiplication of the second intermediate matrix I_2 times the second transposed matrix $FB_O{}^T$.

The mask multiplier 156 produces the third intermediate matrix I_3 as an 8X8 matrix by mask multiplication of the forward DECT matrix S(v,u) with the forward DOCT matrix H(v,u).

The IDCT processor 160 includes a cosine processor 162, a transposer 164 and a matrix multiplier 166. The cosine processor 162 generates an 8X8 IDCT basis matrix IB from the cosine terms of equation (4). A third transposed matrix IB^T is produced as an 8X8 matrix in transposer 164 by transposing the IDCT basis matrix IB. A fourth intermediate matrix I_4 is generated in matrix multiplier 166 as an 8X8 matrix by matrix product multiplication of the IDCT basis matrix IB times the third intermediate matrix I_3. Finally, the second spatial matrix s'(j,i) of reconstructed image data points is generated in the matrix multiplier 166 by matrix product multiplication of the fourth intermediate matrix I_4 times the third transposed matrix IB^T. The second spatial matrix s'(j,i) can be used to print or otherwise reproduce the filtered image. However, if aliasing due to noise is a problem, then overlapping hardware (not shown) could be added to prevent aliasing by the previously discussed methods of overlapping adjacent pixel groups.

Reducing An Image By Decimation

At times it may be desirable to reduce the size of an image for printing or display applications. This can be accomplished by a method referred to as decimation, defined as reducing the image size by eliminating data points. Key features of the above resampling and filtering methods are incorporated into the method of reducing an image by decimation. In fact, reducing an image by decimation is a special case of scaling where the image is scaled down by downsampling using a scaling ratio less than one.

Typically, low pass filtering of an image is performed prior to reducing the image by decimation. The low pass filtering reduces high frequency artifacts. However, pre-decimation filtering is not necessary in the case when the spatial aliasing is inconsequential to accurate image reproduction.

The schematic block diagram of Fig. 9 shows the steps of a method for reducing an image by decimation as described in the following preferred embodiment for two dimensional image data. Repetition of some of the detailed explanations of previously described mathematics and steps will be omitted for the sake of brevity.

Fig. 9 shows a forward DECT section 10, a filtering section 220, and a decimation section 224. The forward DECT section 10 corresponds to the forward DCT section 10 of Fig. 3, the filtering section 220 includes the filtering steps of Fig. 6, and the decimation section is related to the hybrid IDCT section 12 of Fig. 3.

The two dimensional DECT matrix S(v,u), generated as described for the example of enlarging an image by interpolation in Fig. 3, is mask multiplied in the mask multiplier 144 with the forward DOCT matrix H(v,u) to generate a third intermediate matrix I_3 (i.e. a mask multiplied matrix) as previously described for Fig. 6. Decimation section 224 downsamples the two dimensional hybrid IDCT basis matrix in either one or both dimensions so that fewer reconstructed image data points s'(y,x) will be generated than the original number of image data points s(j,i). The reduced image (not shown) is then printed, processed or otherwise reconstructed from the reconstructed image data points s'(y,x).

For the preferred embodiment of a method for reducing an image by decimation as shown in Fig. 9, the forward DECT section 10 includes the step of selecting the first spatial matrix s(j,i) as an 8X8 matrix in block 20. The 8X8 forward DECT basis matrix FB_E is determined in block 22 from the cosine terms of equation (3). In multiplier 25, the 8X8 first intermediate matrix I_1 is generated by matrix product multiplication of the forward DCT basis matrix FB_E times the first spatial matrix s(j,i). The first transpose matrix FB_E^T is generated as an 8X8 matrix in block 24 by transposing the forward DECT basis matrix FB_E. Multiplier 26 produces the 8X8 forward DECT matrix S(v,u) by matrix product multiplication of I_1 times the first transposed matrix FB_E^T. For this

example, the values of the first spatial matrix s(j,i), the forward DECT basis matrix FB_E, the first intermediate matrix I_3, the first transpose matrix FB_E^T, and the forward DECT matrix S(v,u) are the same as those listed above for the example of Fig. 3.

The filtering section 220 includes the step of selecting in block 124 a filter kernel or convolution kernel represented as h(j,i) in the spatial domain. The kernel is selected according to predetermined criteria such as a desire to sharpen or smooth the image. For our example, the following odd, symmetric 3X3 kernel h(j,i) was arbitrarily selected for low pass filtering.

KERNEL, h(j,i)

0.08	0.08	0.08
0.08	0.33	0.08
0.08	0.08	0.08

The 8X8 padded kernel matrix h(j,i) shown below is generated in block 122 to conform with the size of the 8X8 first spatial matrix chosen in block 20.

PADDED KERNEL MATRIX, $h_P(j,i)$

0.33	0.08	0.00	0.00	0.00	0.00	0.00	0.00
0.08	0.08	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The following 8X8 forward DOCT basis matrix FB_O of padded kernel matrix $h_P(j,i)$ is derived from the cosine terms of equation (9) as previously described in the filtering example of Fig. 6.

FORWARD DOCT BASIS MATRIX, FB_O

1.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
1.000	1.848	1.414	0.765	-0.000	-0.765	-1.414	-1.848
1.000	1.414	-0.000	-1.414	-2.000	-1.414	0.000	1.414
1.000	0.765	-1.414	-1.848	0.000	1.848	1.414	-0.765
1.000	-0.000	-2.000	0.000	2.000	-0.000	-2.000	0.000
1.000	-0.765	-1.414	1.848	-0.000	-1.848	1.414	0.765
1.000	-1.414	0.000	1.414	-2.000	1.414	-0.000	-1.414
1.000	-1.848	1.414	-0.765	0.000	0.765	-1.414	1.848

In multiplier 128, the 8X8 forward DOCT basis matrix FB_O is matrix product multiplied times the 8X8 padded kernel matrix $h_P(j,i)$ to produce the 8X8 second intermediate matrix I_2 shown below.

SECOND INTERMEDIATE MATRIX, I_2

0.500	0.250	0.000	0.000	0.000	0.000	0.000	0.000
0.487	0.237	0.000	0.000	0.000	0.000	0.000	0.000
0.451	0.201	0.000	0.000	0.000	0.000	0.000	0.000
0.397	0.147	0.000	0.000	0.000	0.000	0.000	0.000
0.333	0.083	0.000	0.000	0.000	0.000	0.000	0.000
0.270	0.020	0.000	0.000	0.000	0.000	0.000	0.000
0.215	-0.035	0.000	0.000	0.000	0.000	0.000	0.000
0.179	-0.071	0.000	0.000	0.000	0.000	0.000	0.000

The following 8X8 second transposed matrix FB_O^T is generated by transposing the 8X8 forward DOCT basis matrix FB_O in block 130.

SECOND TRANSPOSED MATRIX, FB_O^T

1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.000	1.848	1.414	0.765	-0.000	-0.765	-1.414	-1.848
2.000	1.414	-0.000	-1.414	-2.000	-1.414	0.000	1.414
2.000	0.765	-1.414	-1.848	0.000	1.848	1.414	-0.765
2.000	-0.000	-2.000	0.000	2.000	-0.000	-2.000	0.000
2.000	-0.765	-1.414	1.848	-0.000	-1.848	1.414	0.765
2.000	-1.414	0.000	1.414	-2.000	1.414	-0.000	-1.414
2.000	-1.848	1.414	-0.765	0.000	0.765	-1.414	1.848

The forward DOCT matrix H(v,u) shown below is generated as an 8X8 matrix in multiplier 136 by matrix product multiplication of the 8X8 second intermediate matrix I_2 times the 8X8 second transposed matrix, FB_O^T.

FORWARD DOCT MATRIX, H(v,u)

1.000	0.962	0.854	0.691	0.500	0.309	0.146	0.038
0.962	0.926	0.823	0.669	0.487	0.306	0.152	0.049
0.854	0.823	0.736	0.605	0.451	0.297	0.167	0.079
0.691	0.669	0.605	0.510	0.397	0.285	0.189	0.125
0.500	0.487	0.451	0.397	0.333	0.270	0.215	0.179
0.309	0.306	0.297	0.285	0.270	0.255	0.242	0.233
0.146	0.152	0.167	0.189	0.215	0.242	0.264	0.279
0.038	0.049	0.079	0.125	0.179	0.233	0.279	0.310

The following 8X8 third intermediate matrix, I_3, is generated in mask multiplier 144 by mask multiplication of the 8X8 forward DECT matrix, S(v,u), with the 8X8 forward DOCT matrix, H(v,u).

THIRD INTERMEDIATE MATRIX, I_3

947.500	10.206	-42.994	11.543	8.750	2.919	0.301	-0.264
-3.430	5.826	-3.971	-8.626	-1.751	2.382	-0.991	-0.015
-23.362	8.602	-53.820	-13.036	10.141	-2.449	0.974	-0.431
-0.355	22.571	6.951	-44.151	-0.206	5.575	-2.830	-0.904
11.250	-2.995	28.641	7.666	-9.075	2.982	0.749	-1.785
-3.129	0.606	-5.847	1.206	-2.725	3.170	-2.874	0.569
-1.134	1.403	-1.576	-0.287	-3.840	2.196	5.910	-10.895
0.506	-0.295	1.853	0.459	2.277	-4.111	4.614	-0.736

At this point, a standard inverse DCT (as in equation (4)) of the above third intermediate matrix I_3 would result in an 8X8 second spatial matrix $s'(j,i)$ of reconstructed image data points. If overlapping of adjacent groups of pixels is deemed necessary to prevent aliasing (as previously discussed in accordance with the filtering examples of Figs. 6 and 7), then the overlap for filtering with the above 3X3 kernel matrix $h(j,i)$ would be $k-1$ or two pixels, amounting to a one pixel wide perimeter area $(k-1)/2$ of the second spatial matrix $s'(j,i)$ to be discarded. The discarded elements of $s'(j,i)$ would include all reconstructed image data points in rows 1, 8 and columns 1, 8. The filtered saved reconstructed matrix $s'_S(j,i)$ would be a 6X6 matrix including 36 reconstructed image data points located in rows 2-7 and columns 2-7. In other words, a one pixel wide perimeter of the low pass filtered version of the second spatial matrix $s'(j,i)$ would be discarded to prevent aliasing.

Rather than immediately performing a standard IDCT on the third intermediate matrix I_3 to generate a full size reconstructed filtered image as discussed above, a reduced image can be obtained when scaling down or downsampling by selecting the first dimension scaling ratio in block 23A to be less than one (i.e. 6:8 where $N' = 6$), and selecting the second dimension scaling ratio in block 23B to be less than one (i.e. 6:8 where $M' = 6$), wherein the first dimension scaling ratio equals the second dimension scaling ratio for the preferred embodiment. Conventionally, selection of the scaling ratios has been limited to values which reduce (or enlarge) an image without undue mathematical

computation. For instance, a scaling ratio of 1:2 is typical, since an 8X8 first spatial matrix s(j,i) of image data points is mathematically related to a 4X4 second spatial matrix of sixteen reconstructed image data points by the integer multiple of two. In contrast, a scaling ratio of 2:5 would be atypical. However, according to a feature of the invention, any scaling ratio can be selected for each dimension (as previously described in the methods of Figs. 3 and 4 for enlarging an image by interpolation). Values of reconstructed image data points of the second spatial matrix s'(y,x) which fall between the indexed locations of the image data points of the first spatial matrix s(j,i) are determined by solving s'(x) for values of x falling within the appropriate ranges for u and v in equation (6). For instance, when selecting a 2:5 resampling ratio for each dimension, the values of s'(x) interpolated to a curve (in the range of 0 to 7) are evaluated at x = 2.5 and x = 5 for the first group of eight pixels, as shown and earlier discussed in relation to Fig. 2C.

When choosing both the first and second dimension scaling ratios as 2:5, the following 2X8 hybrid IDCT basis matrix IB_{1HYB} is generated in block 30 from the cosine terms of equation (6).

HYBRID IDCT BASIS MATRIX, IB_{1HYB}

0.354	0.191	-0.354	-0.462	0.000	0.462	0.354	-0.191
0.354	-0.278	-0.191	0.490	-0.354	-0.098	0.462	-0.416

A 2X8 fourth intermediate matrix I_4 as shown below is generated in multiplier 32 of decimation section 224 by matrix product multiplication of the 2X8 hybrid IDCT basis IB_{1HYB} matrix times the 8X8 third intermediate matrix I_3.

FOURTH INTERMEDIATE MATRIX, I_4

340.816	-7.912	-3.756	27.802	-3.784	2.806	0.759	-2.975
335.834	13.183	-11.445	-15.831	2.292	4.933	-0.365	-4.601

An 8X2 third transposed matrix IB_{1HYB}^T (not shown) is generated in block 28 by transposing the hybrid IDCT basis matrix, IB_{1HYB}^T. Finally in multiplier 33, the following 2X2 second spatial matrix s'(j,i) of reconstructed filtered image data points is generated by matrix product multiplication of the 2X8 fourth intermediate matrix I_4 times the 8X2 third transposed matrix, IB_{1HYB}^T.

SECOND SPATIAL MATRIX, s'(j,i)

109.601	139.698
135.647	109.952

In block 36, the image can be printed or otherwise reproduced using the reconstructed image data points of second spatial matrix, s'(j,i).

The ordering of steps in the above examples is not absolute or unyielding in each case. One of ordinary skill in the art could easily ascertain when the ordering of steps is critical. Also, any number of dimensions of image data points could be processed according to the teachings of the invention, i.e. the above examples using one and two dimensional image data points are exemplary rather than limiting. Additionally, the numerous features of the invention can be combined or separately applied as desired. For instance, an image represented by two dimensional image data could be scaled for reduction in one dimension and enlargement in the other dimension. Finally, the above embodiments of novel methods and devices are preferred examples of the many variations and modifications which would be apparent to one of ordinary skill in the art in keeping with the invention as claimed.

What is claimed is:

1. A method of scaling an image represented by at least one discrete even cosine transform (DECT) matrix of N DECT coefficients for $0 \leq u \leq (N-1)$ where u and N are integers, said method comprising the steps of:

selecting a scaling ratio of N':N where N' is an integer;

generating, for each said DECT matrix, a hybrid inverse DECT basis matrix having N' elements; and

generating, for each said DECT matrix, a reconstructed matrix of N' reconstructed image data points in a spatial domain by performing an inverse DECT of each said DECT matrix in accordance with said inverse DECT basis matrix.

2. The method of claim 1, wherein said reconstructed matrices are generated by the inverse DECT equation:

$$s'(i) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C_u \, S(u) \, \cos\frac{(2i+1)u\pi}{2N}$$

for $0 \leq i \leq (N'-1)$,

where: S(u) represents said DECT coefficients;

s'(i) represents said reconstructed image data points;

i is an integer variable;

$C_u = \frac{1}{\sqrt{2}}$ for $u = 0$; and

$C_u = 1$ for $u \neq 0$,

and said hybrid inverse DECT basis matrix is generated from $\cos\frac{(2i+1)u\pi}{2N}$.

3. The method of claim 1, further comprising a process of filtering the image in a manner similar to a mathematical convolution, said process comprising the steps of:

selecting a kernel in the spatial domain, represented as a kernel matrix, according to a predetermined criterion;

generating a padded kernel matrix of N coefficients in the spatial domain by padding said kernel matrix with zeros;

producing, for each said DECT matrix, an N element discrete odd cosine transform (DOCT) basis matrix;

generating, for each said DECT matrix, a DOCT matrix of N DOCT coefficients by matrix product multiplication of each said DOCT basis matrix times said kernel matrix;

generating, for each said DECT matrix, a filtered matrix by mask multiplication of each said DECT matrix with each corresponding said DOCT matrix; and

generating, for each said DECT matrix, a filtered reconstructed matrix of N′ filtered reconstructed image data points in the spatial domain by performing an inverse DECT of each said filtered matrix in accordance with said inverse DECT basis matrix.

4. The method of claim 3, wherin each said DECT matrix overlaps an adjacent said DECT matrix.

5. The method of claim 4, wherein said kernel matrix comprises k elements in one dimension, and said overlapping DECT matrices overlap by k elements, k being an integer.

6. The method of claim 3, wherein each said DOCT matrix is generated from the DOCT equation:

$$H(u) = 2\sum_{i=0}^{N-1} d_i \, h_P \, (i) \, \cos\frac{iu\pi}{N}$$

for $0 \leq u \leq (N-1)$;

where H(u) represents said DOCT coefficients;

$h_P(i)$ represents said coefficients of the padded kernel matrix;

$h_P(i) = 0$ for $|i| > (k-1)/2$;

$d_i = ½$ for $i=0$; and

$d_i = 1$ for $i=1, 2 \ldots (N-1)$.

7. The method of claim 3, wherein said predetermined criterion comprises one of smearing and sharpening the image.

8. The method of claim 1, wherein the image is reproduced as a scaled image from said reconstructed matrices.

9. An apparatus for scaling an image represented by at least one discrete even cosine transform (DECT) matrix of N DECT coefficients for $0 \leq u \leq (N-1)$ where u and N are integers, said apparatus comprising:

means for selecting a scaling ratio of $N':N$ where N' is an integer;

means for generating, for each said DECT matrix, an inverse DECT basis matrix having N' elements; and

means for generating, for each said DECT matrix, a reconstructed matrix of N' reconstructed image data points in a spatial domain by performing an inverse DECT of each said DECT matrix in accordance with said inverse DECT basis matrix.

10. The apparatus of claim 9, wherein said means for generating the reconstructed matrices comprises means for solving the inverse DECT equation:

$$s'(i) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C_u \, S(u) \cos\frac{(2i+1)u\pi}{2N}$$

for $0 \leq i \leq (N' - 1)$,

where: S(u) represents said DECT coefficients;

s'(i) represents said reconstructed image data points;

i is an integer variable;

$C_u = \frac{1}{\sqrt{2}}$ for u = 0; and

$C_u = 1$ for $u \neq 0$,

and said means for generating an inverse DECT basis matrix comprises means for generating the inverse DECT basis matrix from $\cos\frac{(2i+1)u\pi}{2N}$.

11. The apparatus of claim 9, further comprising means for filtering the image in a manner similar to a mathematical convolution, said filtering means comprising:

means for selecting a kernel in the spatial domain, represented as a kernel matrix, according to a predetermined criterion;

means for generating a padded kernel matrix of N coefficients in the spatial domain by padding said kernel matrix with zeros;

means for producing, for each said DECT matrix, an N element discrete odd cosine transform (DOCT) basis matrix;

means for generating, for each said DECT matrix, a DOCT matrix of N DOCT coefficients by matrix product multiplication of each said DOCT basis matrix times said kernel matrix;

means for generating, for each said DECT matrix, a filtered matrix by mask multiplication of each said DECT matrix with each corresponding said DOCT matrix; and

means for generating, for each said DECT matrix, a filtered reconstructed matrix of N' filtered reconstructed image data points in the spatial domain by performing an inverse DECT of each said filtered matrix in accordance with said inverse DECT basis matrix.

12. The apparatus of claim 11, further comprising means for overlapping each said DECT matrix with an adjacent said DECT matrix.

13. The apparatus of claim 12, wherein said means for overlapping comprises means for overlapping by k elements, wherein k is an integer.

14. The apparatus of claim 11, wherein said means for generating the DOCT matrix comprises means for solving the DOCT equation:

$$H(u) = 2\sum_{i=0}^{N-1} d_i \, h_P \, (i) \, \cos\frac{iu\pi}{N}$$

for $0 \leq u \leq (N-1)$;

where $H(u)$ represents said DOCT coefficients;

$h_P(i)$ represents said coefficients of the padded kernel matrix;

$h_P(i) = 0$ for $|i| > (k-1)/2$;

$d_i = ½$ for $i=0$; and

$d_i = 1$ for $i=1, 2 \ldots (N-1)$.

15. The apparatus of claim 11, wherein said predetermined criterion comprises one of smearing and sharpening the image.

16. The apparatus of claim 9, further comprising means for reproducing a scaled said image from said reconstructed matrices.

17. A method of scaling an image represented in a spatial domain as a first matrix of image data points, said method comprising the steps of:

(a) producing a discrete even cosine transform (DECT) matrix of N DECT coefficients in a DECT domain, N being an integer, by a forward DECT of said first matrix, said forward DECT facilitated by matrix product multiplication of a forward DECT basis matrix times said first matrix;

(b) selecting a scaling ratio N':N where N' is an integer;

(c) generating, for each said forward DECT matrix, an inverse DECT basis matrix having N' elements; and

(d) generating, for each said DECT matrix, a reconstructed matrix of N' reconstructed image data points in said spatial domain by performing an inverse DECT of each said DECT matrix in accordance with said inverse DECT basis matrix.

18. The method of claim 17, wherein said inverse DECT of step (d) is represented as:

$$s'(i) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C_u \; S(u) \; \cos\frac{(2i+1)u\pi}{2N}$$

for $0 \leq i \leq (N-1)$,

where: S(u) represents said forward DECT coefficients;

s'(i) represents said reconstructed image data points;

$C_u = \frac{1}{\sqrt{2}}$ for u = 0; and

$C_u = 1$ for $u \neq 0$,

and said inverse DECT basis matrix is generated from $\cos\frac{(2i+1)u\pi}{2N}$.

19. The method of claim 17, further comprising a step for reproducing a scaled said image from said reconstructed matrices.

20. The method of claim 17, further comprising a process of filtering the image in a manner similar to a mathematical convolution, said process comprising the steps of:

selecting a kernel in the spatial domain, represented as a kernel matrix, according to a predetermined criterion;

generating a padded kernel matrix of N coefficients in the spatial domain by padding said kernel matrix with zeros;

producing, for each said DECT matrix, an N element discrete odd cosine transform (DOCT) basis matrix;

generating, for each said DECT matrix, a DOCT matrix of N DOCT coefficients by matrix product multiplication of each said DOCT basis matrix times said kernel matrix;

generating, for each said DECT matrix, a filtered matrix by mask multiplication of each said DECT matrix with each corresponding said DOCT matrix; and

generating, for each said DECT matrix, a filtered reconstructed matrix of N′ filtered reconstructed image data points in the spatial domain by performing an inverse DECT of each said filtered matrix in accordance with said inverse DECT basis matrix.

21. The method of claim 20, wherein each said DECT matrix overlaps an adjacent said DECT matrix.

22. The method of claim 21, wherein said kernel matrix comprises k elements in one dimension, and said overlapping DECT matrices overlap by k elements, k being an integer.

23. The method of claim 20, wherein each said DOCT matrix is generated from the DOCT equation:

$$H(u) = 2\sum_{i=0}^{N-1} d_i \, h_P(i) \cos\frac{iu\pi}{N}$$

for $0 \leq u \leq (N-1)$;

where H(u) represents said DOCT coefficients;

$h_P(i)$ represents said coefficients of the padded kernel matrix;

$h_P(i) = 0$ for $|i| > (k-1)/2$;

$d_i = ½$ for $i=0$; and

$d_i = 1$ for $i=1, 2 \ldots (N-1)$.

24. The method of claim 20, wherein said predetermined criterion comprises one of smearing and sharpening the image.

25. A process of filtering an image represented as a matrix of N image data points, N being an integer, in a spatial domain in a manner similar to a mathematical convolution, said process comprising the steps of:

generating an N element discrete even cosine transform (DECT) matrix of said matrix of image data points by matrix product multiplication of said matrix of image data points times a DECT basis matrix;

selecting a kernel in the spatial domain, represented as a kernel matrix, according to a predetermined criterion;

generating a padded kernel matrix of N coefficients in the spatial domain by padding said kernel matrix with zeros;

producing, for each said DECT matrix, an N element discrete odd cosine transform (DOCT) basis matrix;

generating, for each said DECT matrix, a DOCT matrix of N DOCT coefficients by matrix product multiplication of each said DOCT basis matrix times said kernel matrix;

generating, for each said DECT matrix, a filtered matrix by mask multiplication of each said DECT matrix with each corresponding said DOCT matrix; and

generating, for each said DECT matrix, a filtered reconstructed matrix of N′ filtered reconstructed image data points in the spatial domain by performing an inverse DECT of each said filtered matrix in accordance with said inverse DECT basis matrix.

26. The process of claim 25, wherin each said DECT matrix overlaps an adjacent said DECT matrix.

27. The process of claim 26, wherein said kernel matrix comprises k elements in one dimension, and said overlapping DECT matrices overlap by k elements, k being an integer.

28. The process of claim 25, wherein each said DOCT matrix is generated from the DOCT equation:

$$H(u) = 2\sum_{i=0}^{N-1} d_i \, h_P \, (i) \, \cos\frac{iu\pi}{N}$$

for $0 \leq u \leq (N-1)$;

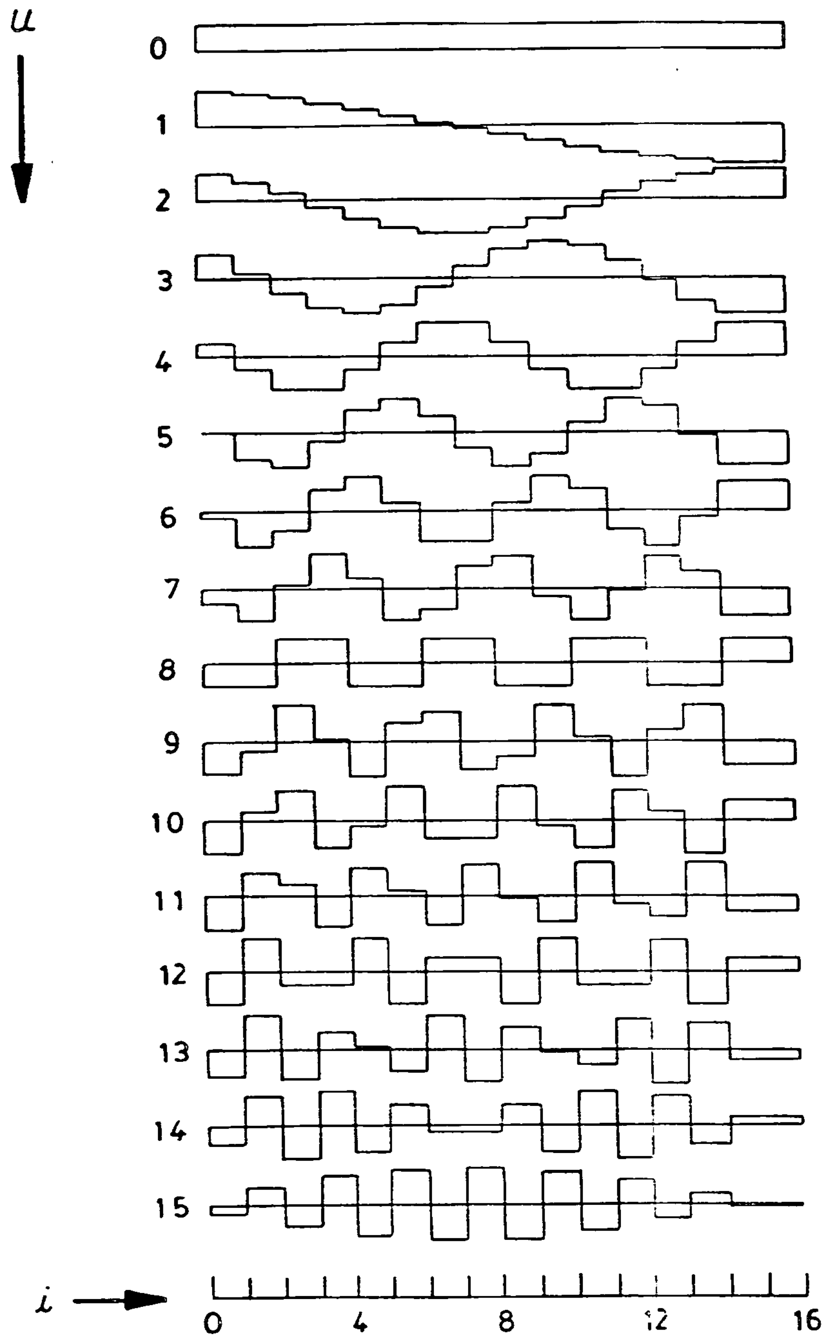
where H(u) represents said DOCT coefficients;

$h_P(i)$ represents said coefficients of the padded kernel matrix;

$h_P(i) = 0$ for $|i| > (N-1)/2$;
$d_i = ½$ for $i=0$; and
$d_i = 1$ for $i=1, 2 \ldots (N-1)$.

29. The process of claim 25, wherein said predetermined criterion comprises one of smearing and sharpening the image.

30. The process of claim 25, wherein said filtered image is reproduced from said filtered reconstructed matrices.



PRIOR ART

FIG. 1

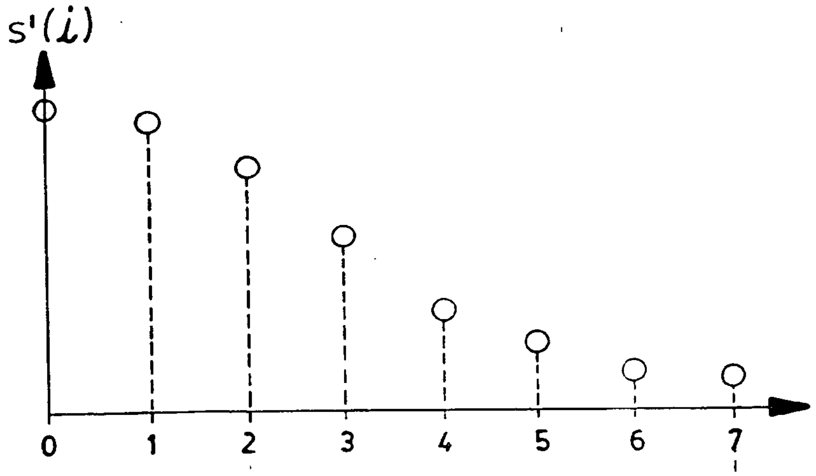


FIG. 2A
PRIOR ART

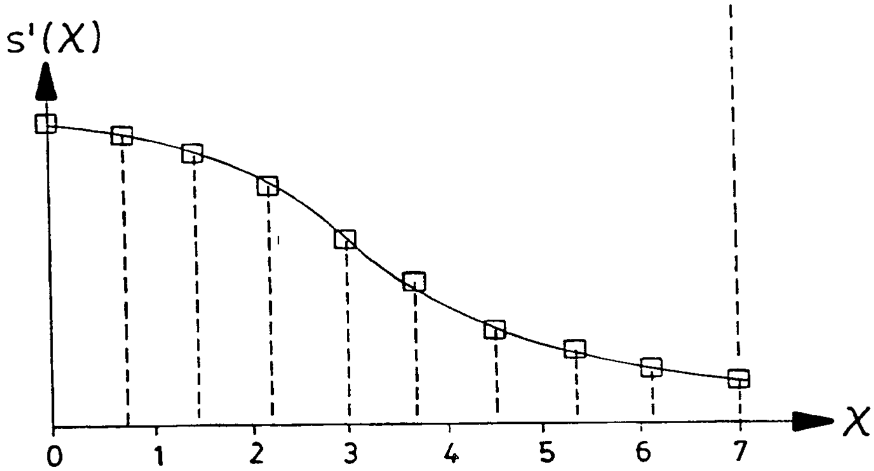


FIG. 2B

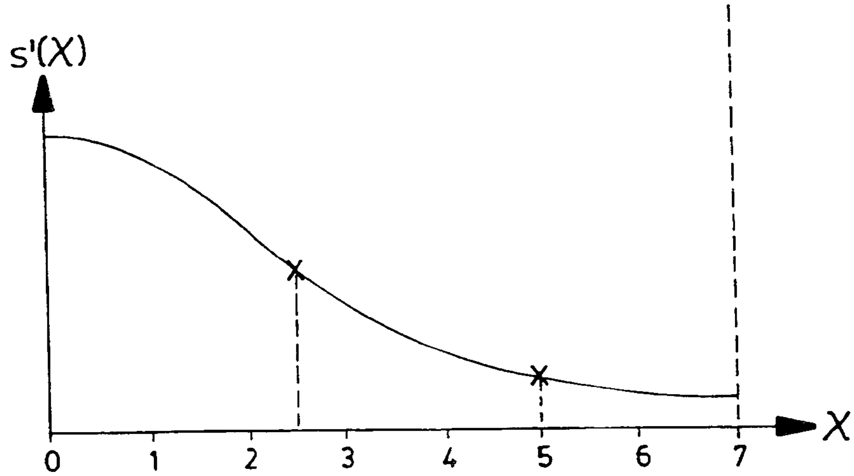


FIG. 2C

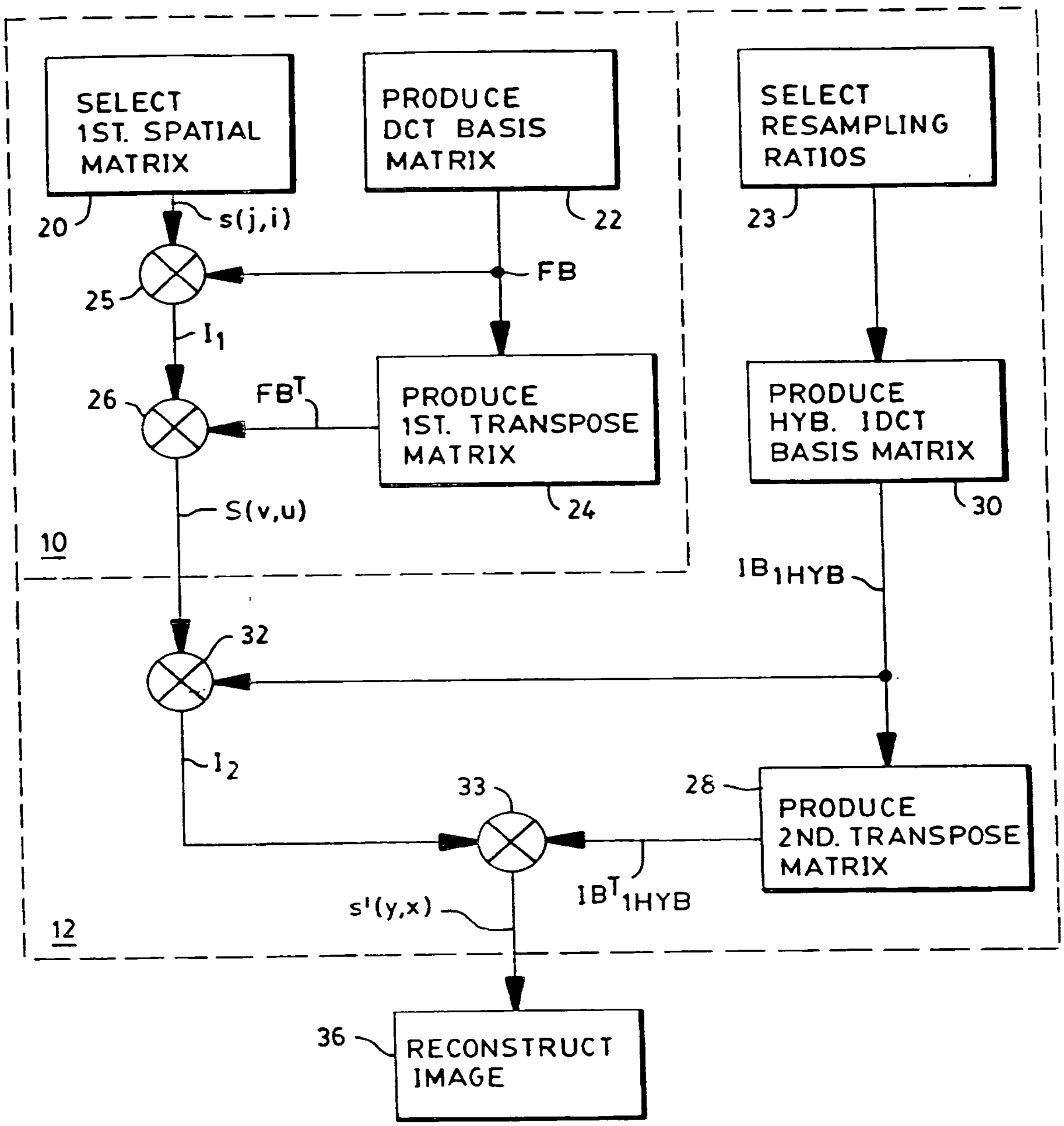
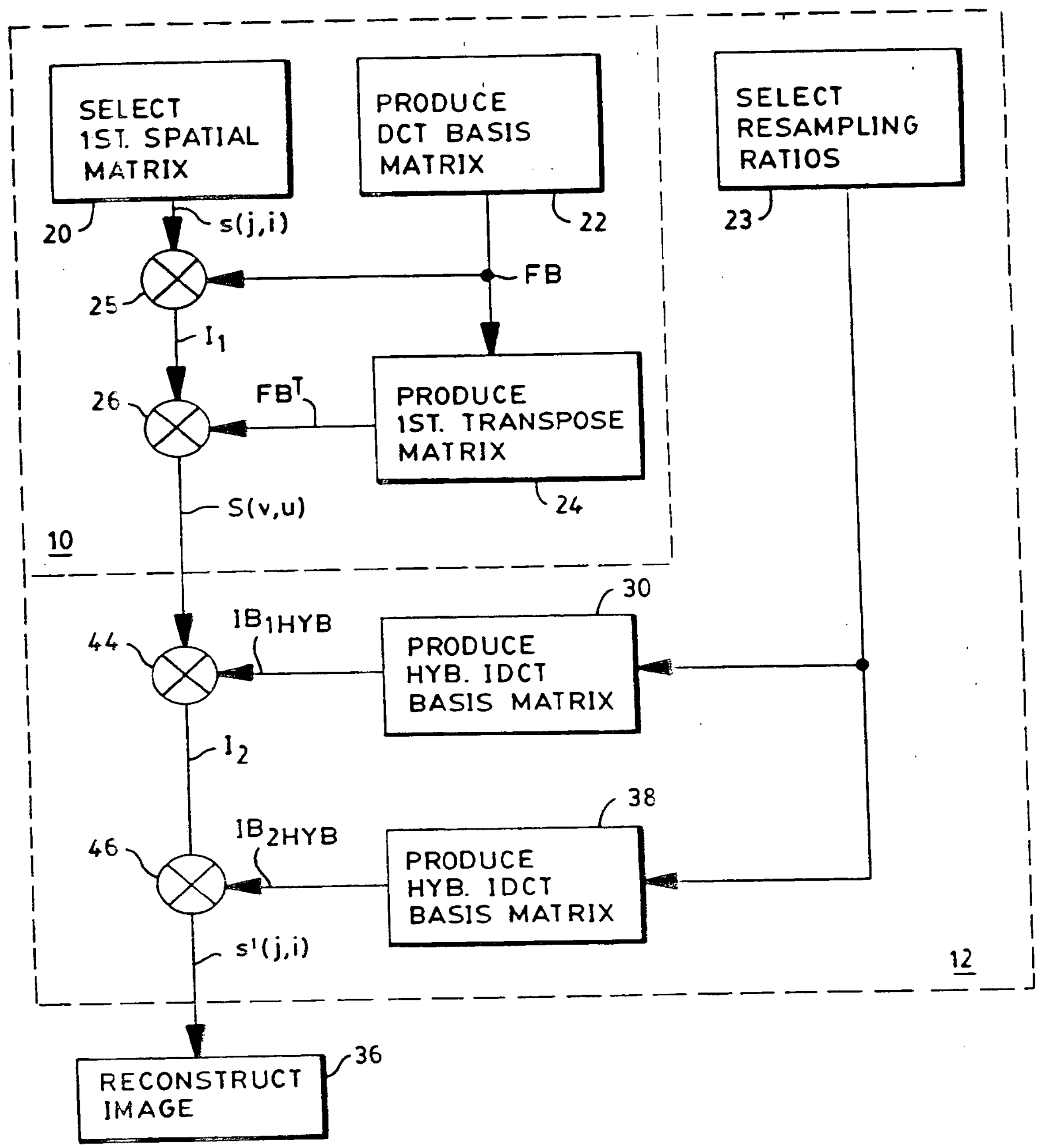

SELECT 1ST. SPATIAL MATRIX
PRODUCE DCT BASIS MATRIX
SELECT RESAMPLING RATIOS
20
s(j,i)
22
23
25
FB
I1
26
FBT
PRODUCE 1ST. TRANSPOSE MATRIX
PRODUCE HYB. IDCT BASIS MATRIX
S(v,u)
24
30
10
IB1HYB
32
I2
33
28
PRODUCE 2ND. TRANSPOSE MATRIX
s'(y,x)
IBT1HYB
12
36
RECONSTRUCT IMAGE

FIG. 3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1ST. SPATIAL
MATRIX
PRODUCE
DCT BASIS
MATRIX
SELECT
RESAMPLING
RATIOS
20
s(j,i)
22
23
FB
25
I1
26
FBT
PRODUCE
1ST. TRANSPOSE
MATRIX
S(v,u)
24
10
30
IB1HYB
44
PRODUCE
HYB. IDCT
BASIS MATRIX
I2
38
46
IB2HYB
PRODUCE
HYB. IDCT
BASIS MATRIX
s'(j,i)
12
RECONSTRUCT
IMAGE
36

FIG.4

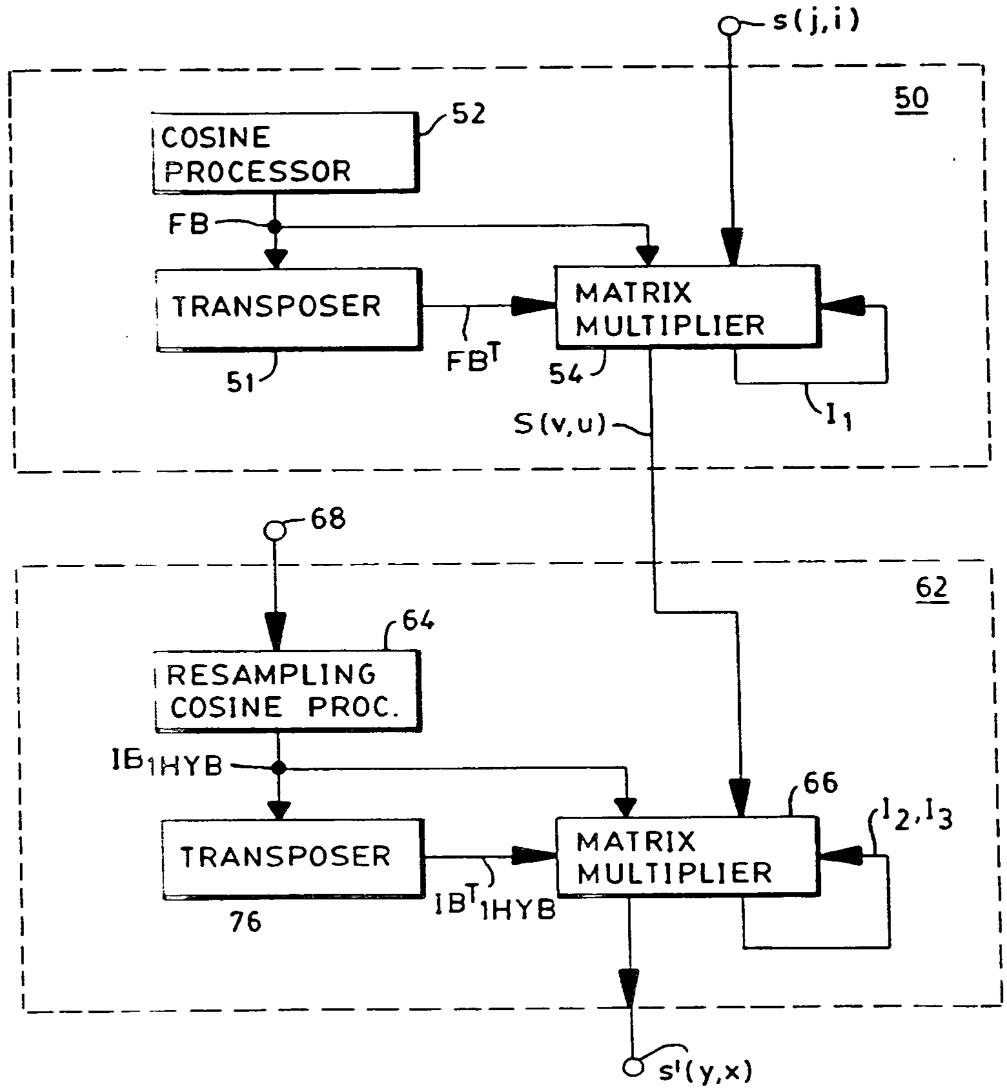
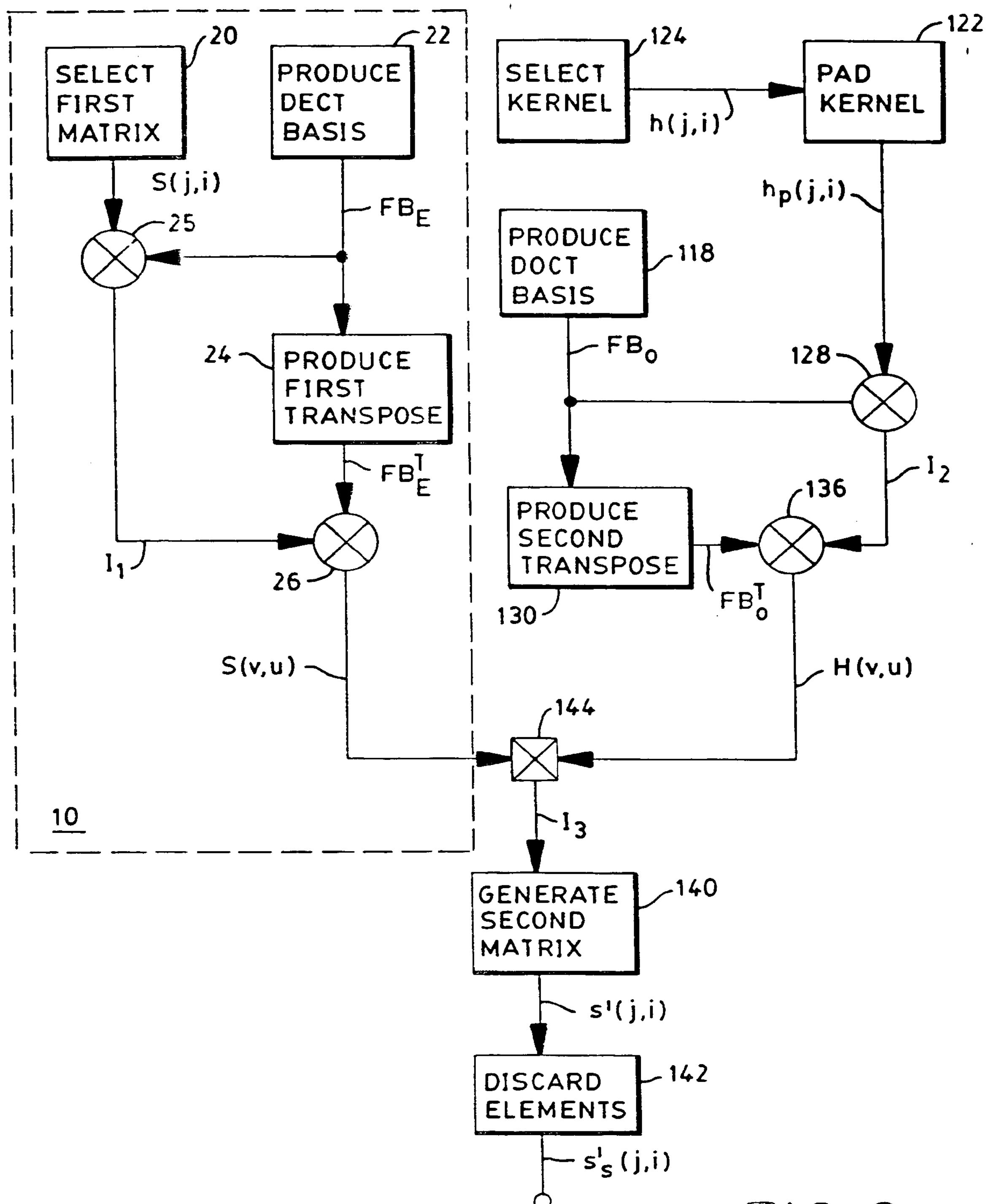

s(j,i)
50
COSINE PROCESSOR
52
FB
TRANSPOSER
51
FB^T
MATRIX MULTIPLIER
54
I1
S(v,u)
68
62
RESAMPLING COSINE PROC.
64
IB1HYB
TRANSPOSER
76
IB^T1HYB
MATRIX MULTIPLIER
66
I2,I3
s'(y,x)

FIG. 5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PRODUCE DECT BASIS
22
S(j,i)
FB_E
25
24
PRODUCE FIRST TRANSPOSE
FB_E^T
I_1
26
S(v,u)
10
SELECT KERNEL
124
h(j,i)
PAD KERNEL
122
h_p(j,i)
PRODUCE DOCT BASIS
118
FB_o
128
I_2
136
PRODUCE SECOND TRANSPOSE
130
FB_o^T
H(v,u)
144
I_3
GENERATE SECOND MATRIX
140
s'(j,i)
DISCARD ELEMENTS
142
s'_s(j,i)

FIG. 6

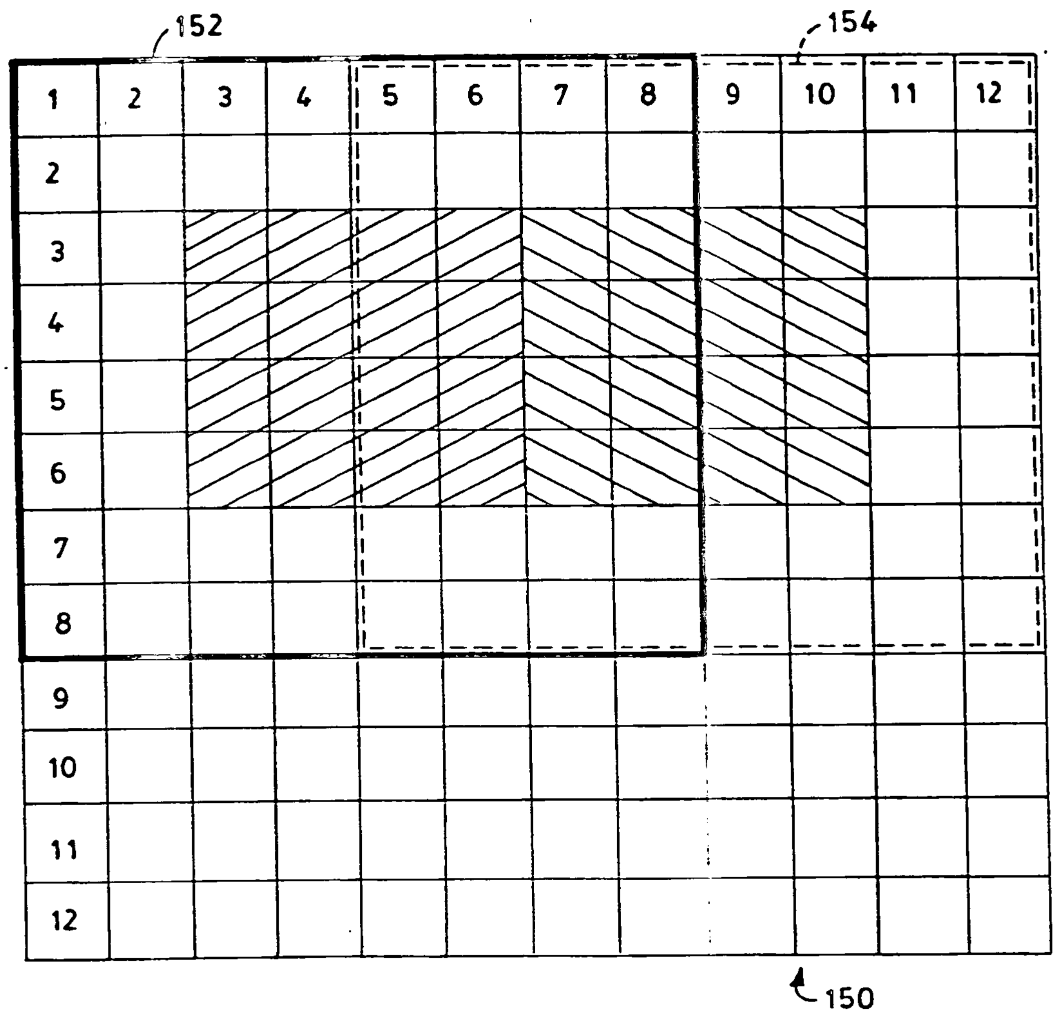
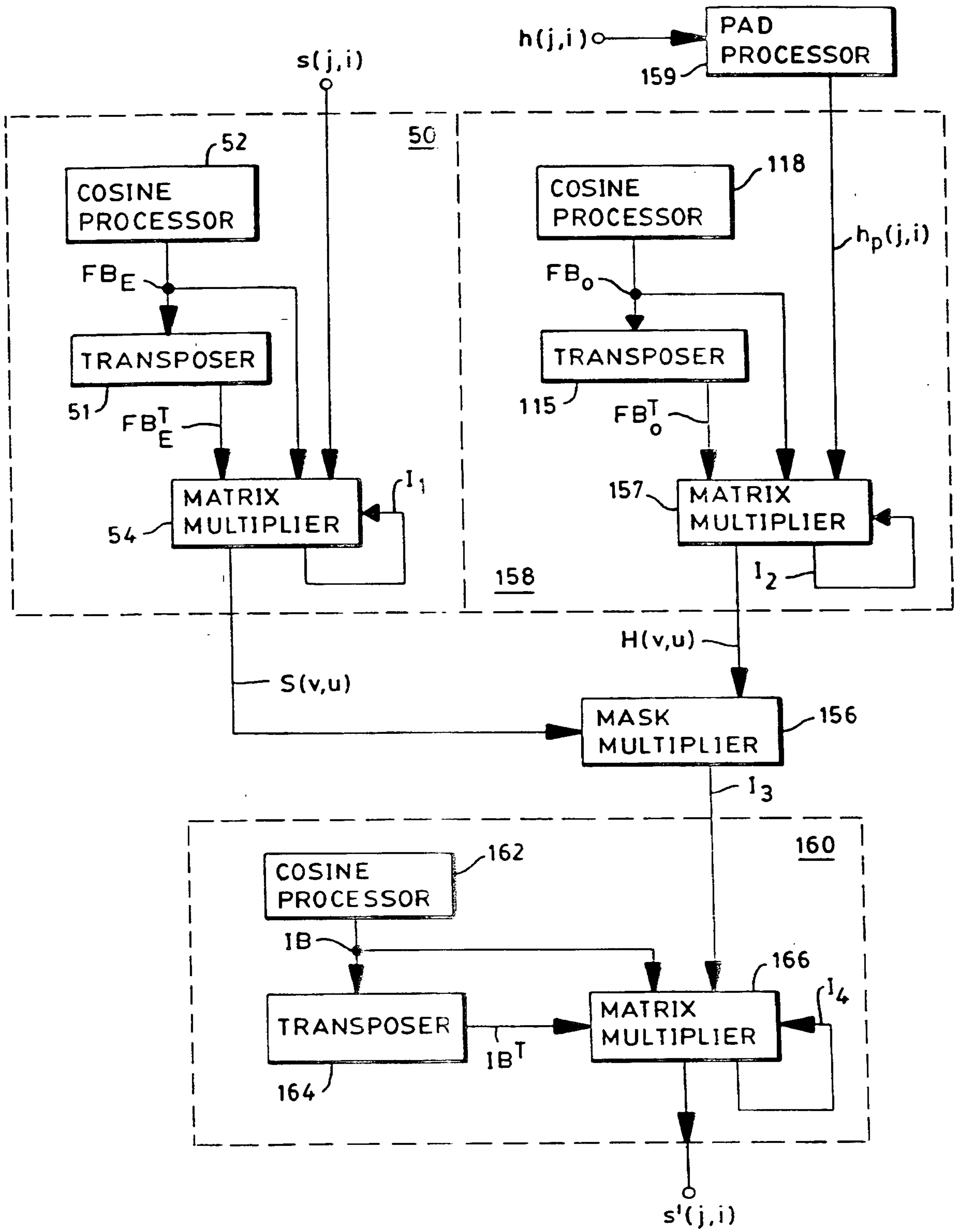


FIG. 7


s(j,i)
h(j,i)
PAD PROCESSOR
159
50
52
COSINE PROCESSOR
FB_E
TRANSPOSER
51
FB_E^T
MATRIX MULTIPLIER
54
I_1
118
COSINE PROCESSOR
$h_p(j,i)$
FB_o
TRANSPOSER
115
FB_o^T
157
MATRIX MULTIPLIER
158
I_2
H(v,u)
S(v,u)
MASK MULTIPLIER
156
I_3
160
COSINE PROCESSOR
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IB
166
I_4
TRANSPOSER
MATRIX MULTIPLIER
IB^T
164
s'(j,i)

FIG. 8

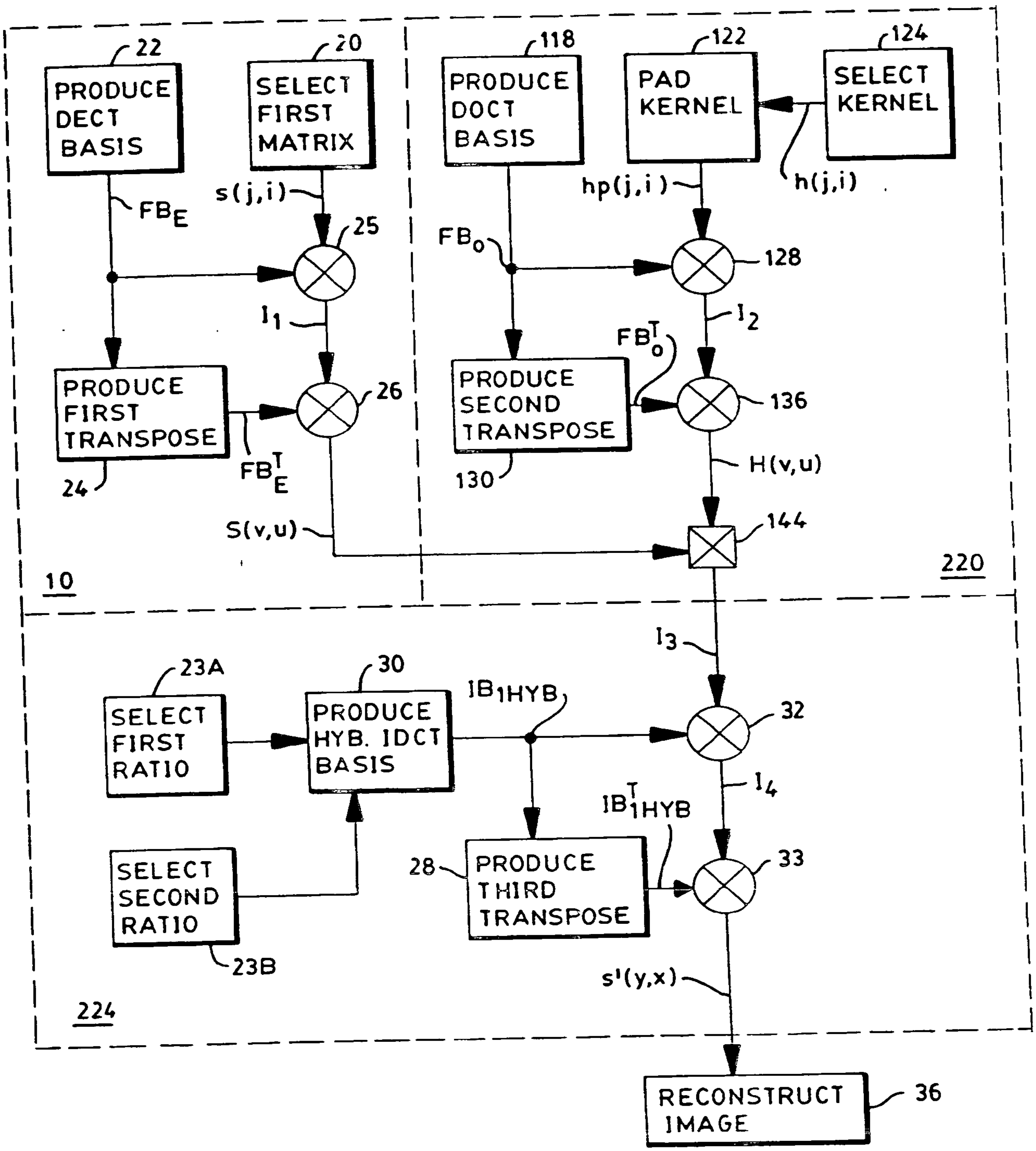
PRODUCE DECT BASIS
22
SELECT FIRST MATRIX
20
PRODUCE DOCT BASIS
118
PAD KERNEL
122
SELECT KERNEL
124
h(j,i)
hp(j,i)
s(j,i)
FB_E
25
FB_o
128
I_1
I_2
FB_o^T
PRODUCE FIRST TRANSPOSE
26
PRODUCE SECOND TRANSPOSE
136
FB_E^T
24
130
H(v,u)
S(v,u)
144
10
220
I_3
23A
30
SELECT FIRST RATIO
PRODUCE HYB. IDCT BASIS
IB_1HYB
32
I_4
IB_1HYB^T
SELECT SECOND RATIO
28
PRODUCE THIRD TRANSPOSE
33
23B
s'(y,x)
224
RECONSTRUCT IMAGE
36

FIG. 9

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US 94/13617

A. CLASSIFICATION OF SUBJECT MATTER
IPC 6 G06T3/40

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 6 G06T G06F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	GB,A,2 211 691 (HITACHI LTD) 5 July 1989 see page 3, line 16 - page 4, line 20; claim 2	1-30
X	IEICE TRANSACTIONS ON FUNDAMENTALS OF ELECTRONICS, COMMUNICATIONS AND COMPUTER SCIENCES, JULY 1993, JAPAN, VOL. E76-A, NR. 7, PAGE(S) 1150 - 1153, ISSN 0916-8508 Muramatsu S et al 'Scale factor of resolution conversion based on orthogonal transforms' see page 1150, right column, paragraph 3 - page 1151, left column, paragraph 2	1-30
A	US,A,5 168 375 (REISCH MICHAEL L ET AL) 1 December 1992 see abstract	1-30

-/--

[X] Further documents are listed in the continuation of box C.

[X] Patent family members are listed in annex.

* Special categories of cited documents :

- "A" document defining the general state of the art which is not considered to be of particular relevance
- "E" earlier document but published on or after the international filing date
- "L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
- "O" document referring to an oral disclosure, use, exhibition or other means
- "P" document published prior to the international filing date but later than the priority date claimed
- "T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
- "X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
- "Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.
- "&" document member of the same patent family

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Date of the actual completion of the international search	Date of mailing of the international search report
13 February 1995	01 03 95

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax (+31-70) 340-3016	Pierfederici, A

Form PCT/ISA/210 (second sheet) (July 1992)

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US 94/13617

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT		
Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	IEEE TRANSACTIONS ON COMPUTERS, JAN. 1974, USA, VOL. C-23, NR. 1, PAGE(S) 90 - 93, ISSN 0018-9340 Ahmed N et al 'Discrete cosine transform' see the whole document ---	2,6,10, 14,18, 23,28
A	PATENT ABSTRACTS OF JAPAN vol. 018, no. 124 (E-1517) 28 February 1994 & JP,A,05 316 357 (NIPPON TELEGR & TELEPH CORP) 26 November 1993 see abstract ---	
A	PATENT ABSTRACTS OF JAPAN vol. 018, no. 025 (M-1542) 14 January 1994 & JP,A,05 261 982 (SEIKO EPSON CORP) 12 October 1993 see abstract -----	

2

INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No.
PCT/US 94/13617

Patent document cited in search report	Publication date	Patent family member(s)		Publication date
GB-A-2211691	05-07-89	JP-A-	1243678	28-09-89
		JP-A-	1114279	02-05-89
		US-A-	5028995	02-07-91
US-A-5168375	01-12-92	NONE		